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Doctoral Thesis

New Approaches to Indirect Demand Response Management in Smart Grids

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Abstract

Demand response has emerged as a solution to shape the load curve and balance the grid. Particularly, indirect demand response management refers to programs based on the load modification of consumer behavior through price signals or incentive payments. In this dissertation, models are proposed to analyze and integrate DR systems in smart grids. The work is addressed in four parts. First of all, a rational behavior of a consumer under uncertainty is quantified in Peak Time Rebate programs. A multi-stage stochastic optimization problem is proposed from the demand side in order to understand the rational decisions. It is found that an optimal consumer alters the consumption patterns in order to increase the profit when there is an economic incentive. Therefore, this kind of programs is vulnerable for gaming. Second, a novel demand response contract between a user and an aggregator is proposed to face gaming concerns. This contract is based on the probability of call, which is the chance of a consumer to be selected by the aggregator to serve as demand response resource at a given period. In this approach, a consumer self-reports his baseline and reduction capacity, given a payment scheme with penalties. A two-stage stochastic programming problem is developed from the demand side to understand the consumer rational decisions under this contract. As results, this solution induces individual rationality (voluntary participation) and asymptotic incentive-compatibility (truthfulness) through the probability of call. Next, another contract for electric vehicles is presented as a solution in the indirect demand response management. A price-based model is proposed to schedule the charging process. A game theoretical model based on Stackelberg structure or bilevel formulation is developed taking into account the interaction between a fleet operator and electric vehicle owners. EV aggregator has the capability to buy energy in the wholesale electricity market and sell it to its customers. The fleet operator objective is to design dynamic prices to consumers that delivering its maximum profit. The proposed price-based scheme presents important results in achieving load shifting and maximizing aggregator profit. Finally, at the market level, a competition between generators with the presence of demand response is proposed. Thermal and hydropower generation are considered in competition within the model. A smooth inverse demand function is designed using a sigmoid and two linear functions for modeling the consumer preferences under incentive-based demand response program. Generators compete to sell energy bilaterally to consumers and system operator provides transmission and arbitrage services. A Nash-Cournot equilibrium is found when the system operates normally and at peak demand times when DR is required. As results, this model shows the effects to employ demand response at the market level, in terms of consumer and producer surplus, and market prices.

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List of Abbreviations

DR	**D**emand **R**esponse
SO	**S**ystem **O**perator
PTR	**P**eak **T**ime **R**ebate
TOU	**T**ime-**o**f-**U**se
EV	**E**lectric **V**ehicle
MPEC	**M**athematical **P**rogramming with **E**quilibrium **C**onstraints
MILP	**M**ixed **I**nteger **L**inear **P**rogramming
KKT	**K**arush-**K**uhn-**T**ucker
TVP	**T**ime-**V**arying **P**ricing
CPP	**C**ritical **P**eak **P**ricing
RTP	**R**eal **T**ime **P**ricing
VCG	**V**icrey-**C**larke-**G**roves
CVaR	**C**onditional **V**alue **a**t **R**isk

Chapter 1

Introduction

Planet Earth is becoming warmer by human actions. Today's society depends largely on fossil fuels for energy, which results in the release of greenhouse gases into the atmosphere producing what is known as global climate change. Electricity contributes significantly to the emission of pollutant gases and also, the energy demand is expected to have a continuous increase in the coming years. Therefore, a significant reduction in the use of fossil fuels must be accompanied by substantial contributions in the electric sector (Varaiya, Wu, and Bialek, 2011). In this sense, new energy models have emerged to face the problem of global climate change. Such models include renewable energies integration, efficient energy conversion, control of emissions, increasing consumer participation (demand response), new energy storage options, deployment of electric vehicles, distributed power sources, etc. The inclusion of these models poses important challenges in the operation of electric grid. Accordingly, smart management systems are required to integrate new paradigms and implement control services and mechanisms to properly handle the supply and demand in the grid.

An important module of the future electric grid, also referred to as smart grid, is the demand side management. Demand response could be employed to induce modified behaviors from consumers to regulate their energy consumption patterns to improve electricity usage. Therefore, an active participation of consumers not only represents a contribution to the carbon emissions reduction but also optimization utilization. There are different ways to active DR in the power systems. Broadly defined, direct control (Diaz, Ruiz, and Patino, 2017; Zhou et al., 2017) and indirect methods (Mohagheghi and Raji, 2015) are found as DR solutions, which are required by the system operator to maintain a fine balance of electricity supply and demand by means of load modification. In particular, indirect control is performed by changing energy prices or giving an incentive payment to participant consumers. For instance, a concrete application of DR can be applied to plug-in electric vehicles in smart charging.

In this dissertation, models are developed to analyze and integrate DR systems in smart grids. This work is addressed in four parts: a model is designed to quantify consumer's rational decisions in a classical incentive-based DR indirect control program called Peak Time Rebate; a contract between consumers and an aggregator is formulated to face gaming and truthfulness concerns associated with this kind of DR service; an aggregator based on the interaction between a fleet operator and EVs under dynamic pricing program is proposed as a price-based DR solution; and finally, a model is developed to analyze competition between generators when incentive-based demand response is employed in electricity markets. Mathematical proofs, numerical studies, and extensive simulations are provided to demonstrate the properties, advantages, and scopes of

these models as solutions and tools in smart grids.
This Chapter reviews some basic aspects of DR systems. First, incentive-based DR is explained by defining the associated features of this programs as well as the disadvantages and challenges in this field. Next, a literature review related to optimal contracts are presented as solutions to the incentive-based DR problems. Furthermore, an introduction to indirect DR methods for EV is described as part of demand-side management. Afterward, an overview of competition in electricity markets with the presence of DR is presented. Finally, thesis contributions and organization are drawn.

1.1 Demand response

In the smart grid concept, DR is a program implemented by SO to equilibrate the load with power generation by modifying consumption. The main purpose of this kind of programs is to curtail load at the peak demand times for maintaining the security of the transmission assets, avoiding to exceed the limit capacity of generators and preventing power outages. Therefore, DR is one of the most crucial parts of the future smart grid (Zhu, Sauer, and Basar, 2013; Bloustein, 2005; Su and Kirschen, 2009) due to the main objectives of DR are to cut the peak, to fill the valley and to shift the load on the power profile. An important question in DR program design is how to improve the demand profile, namely, to control the noncritical loads at the residential, commercial and industrial levels for matching supply and demand. For instance, DR programs might motivate changes in electricity usage by changing consumption patterns, energy price or giving an incentive payment.
There are several DR programs implemented as part of strategies to reduce peak power (because the demand trend is growing). (Vardakas, Zorba, and Verikoukis, 2015; Deng et al., 2015; Siano, 2014; Albadi and El-Saadany, 2008; Madaeni and Sioshansi, 2013; Aghaei and Alizadeh, 2013; Khajavi, Abniki, and Arani, 2011; Palensky and Dietrich, 2011) are surveys that show a complete summary regarding mathematical models, pricing methods, optimization formulation and future extensions. The common approach is TVP, which charges more money for energy usage during peak periods. In TVP program, the consumer does not have a significant incentive to curtail the consumption, just the energy is more expensive at certain hours. Others programs have been implemented where the user behavior is modified through economic incentives, therefore, many utilities have employed a change in the residential electricity rate structure (Newsham and Bowker, 2010). For instance, TOU (Datchanamoorthy et al., 2011; Goudarzi, Hatami, and Pedram, 2011; Hatami and Pedram, 2010; Mohagheghi and Raji, 2015) program, where the day is divided into adjoining blocks of hours. The price of energy varies between blocks, but not within blocks; CPP (Herter, 2007), is related to TOU, unlike that, it is only applied to a small number of event days; RTP (Bloustein, 2005; Allcott, 2012), the price varies hourly according to the real-time market cost of delivering electricity; Direct load method (Ericson, 2009), remote control of flexible loads; Emergency demand reduction (Tyagi and Black, 2010), users receive incentive for diminishing energy consumption during emergency events; PTR (Mohajeryami, Doostan, and Schwarz, 2016; Severin Borenstein, 2014), where customers receive electricity bill rebates by not consuming (relative to a baseline) during peak periods and many other programs. Particularly, PTR belongs to the so-called incentive-based DR programs and

this subject is addressed in this work since PTR solutions exhibit some gaming concerns. This kind of programs is explained below.

1.1.1 Incentive-based demand response

In incentive-based DR, participating agents are paid for diminishing their energy consumption from an established baseline (e.g. PTR, Interruptible Capacity Program and Emergency DR). There are three key components of an incentive-based DR program: 1) A baseline, 2) A payment scheme and 3) Terms and conditions (such as penalties) (Muthirayan et al., 2016). The baseline is defined as an estimation of the energy usage that would have been consumed by demand in the absence of DR (Deng et al., 2015) (see Fig. 1.1). This quantity is often based on the historical consumption of a consumer or a customer group on days that are similar to the forthcoming DR event. Therefore, a counter-factual model is developed to estimate the customer baseline.

The current methods for establishing the baseline are averaging techniques or regres-

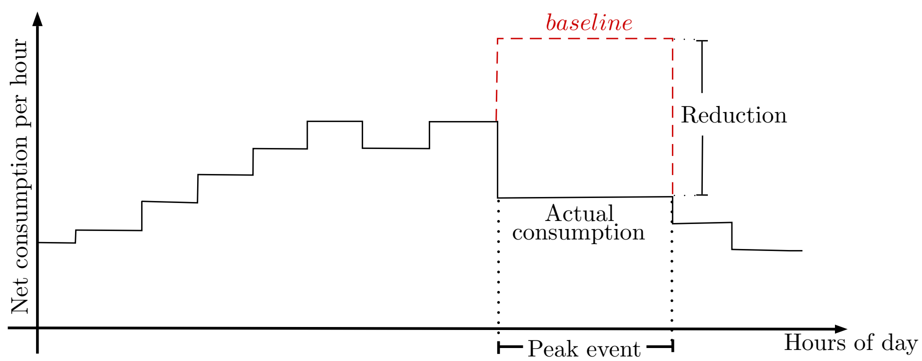

FIGURE 1.1: Baseline definition.

sion approaches. In (Mohajeryami, Doostan, and Schwarz, 2016), some methods are presented to estimate the customer baseline. The performance of DR baselines are studied and new methods are proposed to obtain a reasonable compensation for consumers in (Faria, Vale, and Antunes, 2013; Wijaya, Vasirani, and Aberer, 2014; Antunes, Faria, and Vale, 2013). Baseline model error associated with DR parameter estimates are studied in (Mathieu, Callaway, and Kiliccote, 2011). In addition, in (Chao, 2011), the critical facts on the selection of customer baseline are highlighted, showing that counter-factual forecasts are vulnerable for gaming and could result in illusory demand reduction, then author proposes a baseline focusing on administrative and contractual approaches in order to get an efficient DR. Accordingly, the practice (Severin Borenstein, 2014) has exhibited that consumers have incentives to alter their consumption patterns (gaming behaviors) and baseline setting in order to increase their profit, see e.g. (Zhou, Dahleh, and Tomlin, 2017; Muthirayan et al., 2016). Consequently, inaccurate baselines can derive in over-payment, compromising the cost-effectiveness of the DR program, or in under-payment, negatively affecting the participation of consumers in DR program. Thus one fundamental problem for incentive-based DR programs is how to establish correctly the baseline.

1.1.2 Mechanism design for an incentive-based demand response

Given the fact that the traditional incentive-based DR programs presents gaming concerns when baseline estimation is employed as part of the computation, a mechanism design or a contract could be employed to address these problems in order to guarantee that any participant agent reveals his truthful baseline and private information. Some solutions for DR are found in the literature by designing agreements between consumers and aggregators. In (Ma, Parkes, and Robu, 2017), a truth-telling contract is designed that uses a reward-bidding approach where the mechanism adopts a fixed penalty for non-response for all selected agents, and consumers are selected in increasing order of their minimum acceptable rewards given the penalty. A model of consumer behavior in response to incentives is proposed in a mechanism design framework in (Zhou, Dahleh, and Tomlin, 2017), where aggregator collects the price elasticities of the demand as bids and then it selects the users most susceptible to incentives such that an aggregate reduction is obtained. In (Fahrioglu and Alvarado, 2001), truthful contracts are designed for DR, which maximize the utility benefit function subject to individual rationality and incentive-compatible constraints. A VCG mechanism applied in DR is presented in (Samadi et al., 2012), where the authors verify some properties such as efficiency, user truthfulness, and nonnegative transfer. However, there are several major obstacles implementing VCG mechanisms, see e.g. (Roughgarden, 2016). Other work of VCG approach is found in (Meir, Ma, and Robu, 2017). In (Chen et al., 2012), a game theoretic DR strategy is developed, which consists of a distributed load prediction system by the participation of users that guarantee cheat-proof (truth-telling) behavior. Furthermore, in (Dobakhshari and Gupta, 2016; Dobakshari and Gupta, 2016), a contract between a customer and an aggregator for incentive-based DR is proposed. This mechanism is composed of two parts: a share of aggregator profit and a compensation paid to customer due to load reduction. The above literature overview does not contemplate a self-reported baseline model. Nevertheless, in (Muthirayan et al., 2016; Muthirayan et al., 2017), an incentive-based DR mechanism is proposed, where each consumer reports his baseline consumption and his marginal utility to the aggregator by assuming a linear utility function for each user. Furthermore, individual rationality is demonstrated. However, some concerns raise regarding to the model and implementation. In economics, utility curve is usually modeled as strictly concave function (Vega Redondo, 2003) to depicts consumer's preference. In addition, from the implementation viewpoint, marginal utility information could be a difficult parameter to bid by a user because it is an abstract concept and it could not be easy to estimate by a regular consumer.

1.1.3 Price-based solutions for electric vehicle operators

Electric vehicles represent new issues for the approach of DR programs. EVs have become integral parts of the current grid. For instance, taxi services and urban delivery companies have introduced EVs in their fleet (Bizzarri et al., 2016). However, on the distribution level, the additional loads created by the growing number of EV may have negative impacts on the grid due to undesirable conditions during the charging process (Gruosso, 2017a; Gruosso, 2017b). This fact poses new challenges to the power system operation in terms of smart charging management by fleet operators. The aim of the aggregator is to address two different problems. The former is the price determination in order to satisfy both "user" and "aggregator" needs, the latter is to induce in the EV user a charging behavior useful to take into account the need of the grid in term of production cost and overload. In order to do this it is important to schedule the charging of EVs.

Broadly speaking, there are two ways to schedule the charging process: direct control and indirect methods. The first one means that aggregator schedules directly the charging profile of each EV. In this model, better results are obtained since by being a centralized solution since power system security is guaranteed. However, the fleet operator needs bidirectional communication and smart devices with EVs, thus, an important investment is required to deploy a proper infrastructure. On the other hand, indirect methods are based on price/incentive signals or market solutions in order to influence the consumer behavior (EV owners). The main advantage of this approach is that the infrastructure cost can be reduced. Nevertheless, the solution is not necessarily optimal since depends on the used method and the quality of demand model (Hu et al., 2016).

For case of indirect control, electricity price can be properly formed to diminish load at the peak time periods while increasing EV penetration level in the electricity market (Shao et al., 2010; Faruqui et al., 2011; Yu, Yang, and Rahardja, 2012; Huang et al., 2015). A decentralized charging approach based on TOU tariffs is proposed in (Vaya and Andersson, 2012), where each driver seeks to minimize their costs respecting battery size, charging power level and the driver needs constraints. Similarly, a decentralized computational algorithm is proposed in (Ma, Callaway, and Hiskens, 2013) by focusing on studying Nash equilibrium of the charging problems of EV large population. Furthermore, dynamic price signal is proposed to avoid transformer overload in (Yu, Yang, and Rahardja, 2012). In (Shao et al., 2010), TOU tariffs are studied in the context of EV charging profile. That work claims the importance of designing appropriate rates to motivate demand response. As well, TOU pricing is proposed in (Faruqui et al., 2011), where a social experiment is suggested to estimate the price response of EV charging. Additionally, distribution locational marginal pricing is developed to manage network congestion in (Huang et al., 2015). Nevertheless, more research in price response model is needed to achieve the required performance in the grid ((Hu et al., 2016)).

Related to bilevel approaches or Stackelberg structures, some solutions are found for direct methods of EV charging management. In (Wu et al., 2016), an MPEC formulation is developed for optimal bidding strategies of EV aggregators in day-ahead energy and ancillary services markets with variable wind energy, in which the upper-level problem is the aggregators' Conditional value at risk maximization, while the lower-level problem represents the system operation cost minimization. Additionally, in (Gonzalez Vaya and Andersson, 2015), an MPEC approach of a price-maker aggregator for bidding into the day-ahead electricity market with the aim of minimizing charging costs while satisfying the EV flexible demand is proposed. In that approach, The upper-level problem corresponds to the charging cost minimization of the aggregator, whereas the lower-level problem represents the market clearing. Furthermore, a stochastic robust optimization formulation is presented in (Baringo and Sánchez Amaro, 2017). The bidding strategy of an EV aggregator that participates in the day-ahead energy market is developed. The model output is bidding curves that are submitted to SO. Additionally, a bilevel programming approach or MPEC between a parking lot and SO is developed in (Aghajani and Kalantar, 2017). In that model, the upper-level problem represents the operation cost minimization from SO while the lower-level problem corresponds to the scheduling energy and reserve with the aim of minimizing the parking cost. Moreover, a profit-maximizing EV aggregator is developed in (Sarker, Dvorkin, and Ortega-Vazquez, 2016) that participates into day-ahead energy and reserve markets as a price-taker agent which includes compensation for battery degradation.

1.1.4 Competition in electricity markets with demand response

The co-existence of a variety of generation technologies is an interesting problem from a gaming viewpoint and even more with the integration of DR into the electricity market. In (Genc and Thille, 2008), the competition between hydro and thermal electricity generators under uncertainty over demand and water flows is presented. The authors in (Garcia, Campos-Nañez, and Reitzes, 2005) analyze the price-formation in an oligopoly model where hydroelectric generators are involved in dynamic Bertrand competition. Furthermore, in (Villar and Rudnick, 2003), a model to understand a hydrothermal electric power market is built based on simple bids to the SO. Moreover, in (Zhu, Sauer, and Basar, 2013), by means of Stackelberg game is illustrated what is the value that DR management can bring to generation companies and consumers in a smart grid. Previous works are models of competition between generators, however, they do not include explicitly the DR problem. In addition, (Yao, Oren, and Adler, 2007; Yao, Adler, and Oren, 2008) introduce models of two-settlement electricity market, which accounts for flow congestion, demand uncertainty, system contingencies, and market power. In (Su and Kirschen, 2009), a method is devised for quantifying the effect of the demand response for the market as a central planner. in this last paper, the demand curve has two parts: perfect inelastic behavior and price responsive consumers. The inconvenience of (Su and Kirschen, 2009) is that demand does no have perfect inelastic role since the consumers have a limited willingness to pay. Therefore, another interesting context is to delve into the effect of DR under competition in electricity markets.

1.2 Organization and thesis contributions

The dissertation is organized in chapters and each one can be read independently. The four main contributions of this dissertation are depicted in Fig. 1.2, numbered by the respective chapters and an abstraction of their roles in smart grids.
In previous Section was explained a kind of DR program which is based on incentives. Literature review and practice describe qualitatively the issues and gaming concerns associated to this DR solutions. The first contribution, identified in Chapter 2, is the quantification of these problems through the study of consumer's strategic behavior that participates in a PTR agreement motivated by the concerns in (Severin Borenstein, 2014). In particular, rational consumer decisions are modeled in this part to measure the deviation of baseline formation. In the economic sense, rational behavior means that the users maximize their profits given the mechanism of demand energy reduction. The optimal decision problem is posed as a stochastic optimization algorithm taking into account several previous periods of setting-time in a PTR program. This formulation is solved backward in time to find the optimal choice for consumers where consumer

uncertainty is modeled as a random variable. A closed form solution of a PTR program is derived for two periods. As result, aforementioned inefficiencies are demonstrated; these results are reporters in papers (Vuelvas and Ruiz, 2015; Vuelvas and Ruiz, 2017).

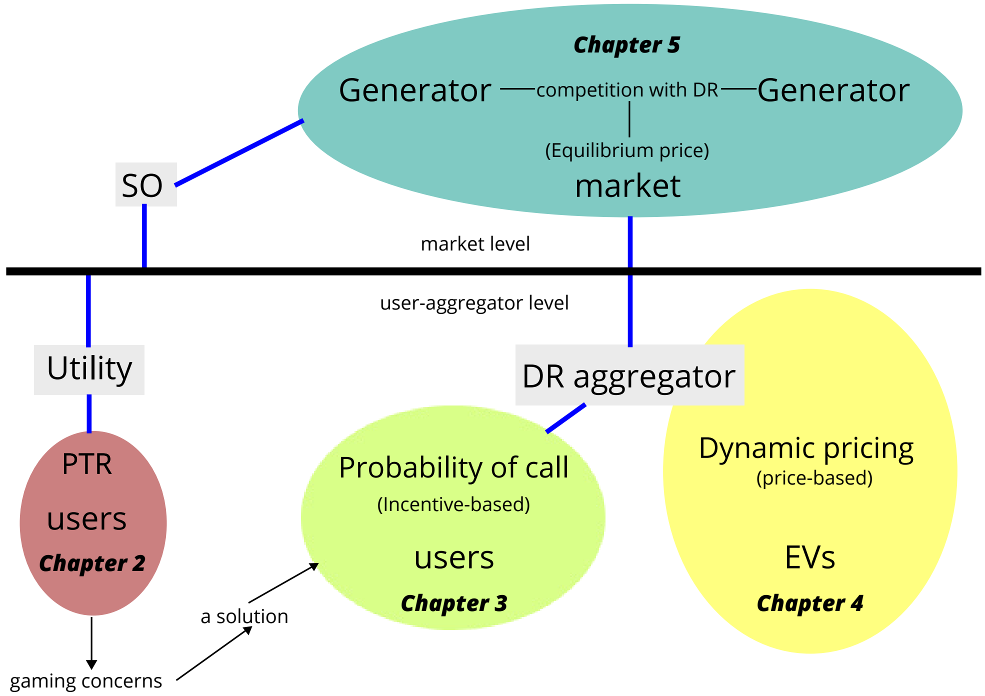

FIGURE 1.2: Contributions.

Given the problems in DR programs associated by employing counter-factual models (Chao, 2011), then new schemes of contracts are required. An implementable solution for the problem described in Chapter 2 is presented through the DR contract of Chapter 3. A new concept is proposed, called the probability of call, to limit the baseline alteration. This contract between a user and an aggregator is developed to induce individual rationality (voluntary participation) and asymptotic incentive-compatibility (truthfulness) through the probability of call, which is the chance of a consumer to be selected by the aggregator to serve as demand response resource at a given period. In this approach, a consumer self-reports his baseline and reduction capacity, given a payment scheme that includes cost of electricity, incentive price, and penalty caused by any deviation between self-reported and actual energy consumption. Therefore, aggregator does no require to estimate/forecast the customer baseline, then it only randomly calls participant consumers with a probability of call close to zero in order to obtain truthful behavior by demand side. Another important feature of this approach, different from the classic solutions described in previous Section, is that a participant agent does not require reporting marginal utility (energy preference), and only announces information in terms of energy, making it easier to implement. In addition, the reduction capacity information gives valuable data to the aggregator in order to plan its aggregated DR capacity for participating in the wholesale electricity market or in bilateral contracts with SO or other agents. The results of this Chapter were derived in the publications (Vuelvas, Ruiz, and Gruosso, 2018a; Vuelvas, Ruiz, and Gruosso, 2018b).

Continuing with agreements at the user-aggregator level, Chapter 4 proposes an aggregator based on the model of the interaction between a fleet operator and electric vehicles under dynamic pricing program. A bilevel optimization problem is formulated to depict the game between the involved agents. At the upper-level, aggregator maximizes its profit whereas the lower-level represents the behavior of rational EV-owners as a fleet. The electric vehicle group is modeled as a virtual battery with forecasted energy demand. Furthermore, the uncertainties are modeled in a scenario-probability framework in the formulation. An advantage of this approach is that EV fleet optimization problem is incorporated in the model avoiding solutions that arbitrarily choose demand elasticities or consumer benefit functions. As results, this approach can be used as price planner for fleet operators in indirect methods of charging management of electric vehicles by sending price-incentive to shift EV demand to periods where the aggregator can obtain better benefits than a fixed price contracts.
Finally, at the market level, Chapter 5 presents a model to analyze of competition between generators when incentive-based demand response is employed in an electricity market. This part considers the same assumptions described in Chapters 2 and 3. Thermal and hydropower generation are considered in the model. A smooth inverse demand function is designed using a sigmoid and two linear functions for modeling the aggregated preferences under incentive-based demand response program. Generators compete to sell energy bilaterally to consumers and system operator provides transmission and arbitrage services. A Nash–Cournot equilibrium is found when the system operates normally and at peak demand times when DR is required. These findings have been published in (Vuelvas and Ruiz, 2018).

The main results can be summarized as follows.

- A model is developed that quantifies rigorously the gaming behavior of consumers that participate in current incentive-based DR programs (Chapter 2). As results, whether the incentive is lower than the retail price, the user shifts his load requirement to the baseline setting period. On the other hand, if the incentive is greater than the regular energy price, the optimal decision is that the user spends the maximum possible energy in the baseline setting period and reduces the consumption at the PTR time. This consumer behavior produces more energy consumption in total considering all periods. In addition, the user with high uncertainty level in his energy pattern should spend less power than a predictable consumer when the incentive is lower than the retail price.

- This part is aimed at the price-based contracts/agreements to incentive DR process. Related to incentive-based DR contract (Chapter 3), the aggregator decides randomly what users are called to perform the energy reduction in order to manage the truth-telling behavior of each agent through the probability of call. This contract induces voluntary participation and asymptotic truthful properties based on the probability criterion. In addition, this approach is susceptible to be implemented since this contract uses information in terms of energy which can be obtained by power monitors. In particular, if an aggregator keeps the probability of call close to zero then consumers reveal their truth information about energy preferences. In addition, regarding to price-based solution for EVs (Chapter 4), an agreement designed from the aggregator viewpoint in order to derive optimal prices that maximize its profit whereas driver-owners are minimizing costs as EV fleet. This relation between agents has hierarchical structure pertaining to the so-called Stackelberg games. The formulation allowed linking the EV fleet decisions and the aggregator objectives to determine the price signal and the optimal load pattern. Subsequently, in the proposed case, the EV fleet operator, while maximizing its profits, sends EV owners a price-incentive to shift their electric charging demand to periods where the aggregator can obtain better benefits than a fixed price contract.

- Finally, this item is concerned with the analysis of DR at the market level (Chapter 5). An analysis of Cournot competition in an incentive-based DR program is investigated. A new demand curve is proposed for modeling consumer preferences in order to include DR in the electricity market. The demand model was devised as a composition of two linear functions and a sigmoid, which represents an energy threshold for analyzing the load reduction in this kind of DR programs. It was found that the incentive-based DR is a cost-effective solution to reduce energy consumption during peak times. However, this program affects negatively the generator surplus under competition environment.

Chapter 2

Rational consumer decisions in a peak time rebate program

In this Chapter, a rational behavior of a consumer is analyzed when an user is enrolled in an incetive-based DR program subject to a baseline or a counter-factual model called PTR. In this DR program, customers receive electricity bill rebates by not consuming during peak periods. In the economic sense, rational behavior means that the users maximize their profits given a contract of demand energy reduction. This rebate is calculated using a baseline for each user which is estimated from past energy consumption. In (Chao, 2011), the critical facts on the selection of customer baseline are highlighted, claiming that counter-factual forecasts are vulnerable for gaming and could result in illusory demand reduction. In real life, the PTR program has shown to be an inefficient DR contract to improve the demand profile because it allows that some users deliberately increase consumption during baseline-setting times (Wolak, 2006; Severin Borenstein, 2014; LLP, 2013). More details about this problem were presented in Subsection 1.1.1. Such consumer behavior of altering the baseline is formulated as a stochastic optimization problem to understand how the users take their decisions of consumption when they are participating in PTR program. While the user intuitively makes decisions according to the operation of the contract, in this Chapter, a mathematical model of consumer choice is proposed in order to quantify the aforementioned inefficiency of the PTR program. These results are reporters in papers (Vuelvas and Ruiz, 2015; Vuelvas and Ruiz, 2017). The key points are described as follows:

- The optimal decision problem is posed in general form taking into account several previous periods of setting-time in a PTR program. The purposed solution is solved backward in time to find the optimal choice for consumers where consumer uncertainty is modeled as a random variable. In addition, the choice of the SO is modeled as a binary random variable, namely, for indicating whether the user is called for participating in PTR mechanism.
- A closed form solution of a PTR program is derived for two periods. The previous consumption is assumed as the baseline and the user is always called to participate in the PTR program. The results show that the consumer alters the baseline when the incentive exists in the DR program. Some numerical examples are presented.

This Chapter is organized as follows. Section 2.1 describes the preliminary setting. In Section 2.2, the general problem formulation of the PTR program is developed. Section 2.3, the mathematical solution for two periods given the optimization problem is explained. Section 2.4, the simulation results are shown. Final comments are drawn in Section 2.5. Lastly, mathematical proofs are presented in Section 2.6.

2.1 Setting

This Section presents the notation and assumptions for developing the model. An individual consumer or aggregated demand (a group of users with the same or similar preferences) is considered for this DR model. the decision maker's preferences are specified by giving utility function $G(q_t;\theta_t)$, where q_t is the consumption at time t and θ_t is a particular realization of random variable Θ. The randomness Θ are external factors that influence the energy requirements of the consumer. The randomness in the utility function is modeled as an additive load requirement, that is, $G(q_t;\theta_t) = G(q_t - \theta_t)$. Θ is assumed to have a probability density function $f_\Theta(\theta_t)$ with limited support $\left[\underline{\theta}, \overline{\theta}\right]$ and mean zero. The motivation to choose such additive randomness is that an external event, such an as a cold wave, will drive the user to increase his energy consumption until he obtains the same comfort than without the event. Then, given a price, the effect of the random event is to shift the equilibrium point to the left in this situation.

The consumer is assumed with risk-averse behavior. Individuals will usually choose with lower risk, therefore, $G(\cdot)$ is concave (Vega Redondo, 2003). This behavior reflects the assumption that marginal utility diminishes as wealth increases. Also, $G(\cdot)$ is considered smooth, positive and nondecreasing.

A competitive electricity market (consumers are price-takers) is assumed. Thus, the energy price p is given and constant since the utility company set an invariable price to the users during the certain period. Then, the following definitions are stated.

Definition 1. *The energy total cost is* $\pi(q_t) = pq_t$.

Definition 2. *The payoff function is defined as* $U_t\left(q_t, \theta_t\right) = G(q_t - \theta_t) - \pi(q_t)$*, which indicates the user benefit of consuming* q *energy during the interval* t.

Definition 3. *Given* $G(\cdot)$, θ_t *and* p*, the rational behavior of the consumer that maximizes the payoff function* $U_t\left(q_t, \theta_t\right)$ *is*

$$q_t^*(\theta_t) = \overline{q} + \theta_t \tag{2.1}$$

this result is found by solving the optimization problem

$$q_t^* = \max_{q_t \in [0, q_{max}]} U_t\left(q_t, \theta_t\right) = G\left(q_t - \theta_t\right) - \pi\left(q_t\right)$$

where q_{max} *is the maximum allowable consumption value,* $\overline{q}$ *is the optimal solution to the previous condition when* $\theta_t = 0$.

2.1.1 Utility function and rebate description

Under assumption that $G(\cdot)$ is a smooth and concave function, the utility function can be approximated by a second order polynomial around $\overline{q}$. Therefore, a quadratic function is considered, where the user utility is zero whether his consumption is zero and saturates after achieving the maximum of the quadratic form, i.,e.,

$$G\left(q_t\right)=\left\{\begin{array}{cc}-\frac{\gamma}{2}\left(q_t-\overline{q}\right)^2+p\left(q_t-\overline{q}\right)+k & 0\leq q_t\leq\overline{q}+\frac{p}{\gamma}\\ -\frac{p^2}{2\gamma}+\frac{p^2}{\gamma}+k & q_t>\overline{q}+\frac{p}{\gamma}\end{array}\right.$$

The saturated part is motivated due to the fact that the agent has a limited well-being with respect to his energy consumption.

Definition 4. *Under additive uncertainty and using the previous consideration (2.1), The utility function can be rewritten as follow:*

$$G\left(q_t-\theta_t\right)=\left\{\begin{array}{cc}-\frac{\gamma}{2}\left(q_t-q_t^*\right)^2+p\left(q_t-q_t^*\right)+k & 0\leq q_t\leq q_t^*+\frac{p}{\gamma}\\ -\frac{p^2}{2\gamma}+\frac{p^2}{\gamma}+k & q_t>q_t^*+\frac{p}{\gamma}\end{array}\right. \tag{2.2}$$

where γ and k are constant. In particular, γ depicts consumer private preferences and k is settled when $G\left(q_t-\theta_t\right)=0$ if $q_t-\theta_t=0$. A similar approach to model a utility function is found in (Samadi et al., 2012). A further discussion about γ can be reviewed in (Fahrioglu and Alvarado, 2001)

Note that γ is in dollar or any other currency divided by energy units squared, therefore, this parameter could be interpreted as the marginal utility that the consumer has as decision-maker into the electricity market. The first order approximation of $\frac{\partial G(q_t-\theta_t)}{\partial q_t}$ when $0\leq q_t\leq q_t^*+\frac{p}{\gamma}$ around q_t^* is

$$\frac{\partial G(q_t-\theta_t)}{q_t}=p-\gamma\left(q_t-q_t^*\right)$$

where $-\gamma$ is the second derivative of $G(\cdot)$ when $0\leq q_t\leq q_t^*+\frac{p}{\gamma}$.

2.1.2 Rebate definition

Basically, DR programs request customers to curtail demand in response to a price signal or economic incentive. Typically the invitation to reduce demand is made for a specific time period. There are three main concepts:

Definition 5. *Baseline: The amount of energy the user would have consumed in the absence of a request to reduce (counterfactual model) (Deng et al., 2015). This quantity can not be measured, then it is estimated from the previous consumption of the agent, i.e., the baseline takes into account $q_{t-1},...,q_{t-n}$. Where n defines the historical consumer behavior, i.e., n corresponds to the periods taken into account within the baseline function.*

$$\text{Baseline}=b(q_{t-1},...,q_{t-n}) \tag{2.3}$$

Definition 6. *Actual Use (q_t): The amount of energy the customer actually consumes during the event period.*

Definition 7. *Load Reduction ($\triangle_t\left(b(\cdot),q_t\right)$): The difference between the baseline and the actual use.*

$$b-q_t=\triangle_t\left(b(q_{t-1},...,q_{t-n}),q_t\right)$$

In PTR programs, the rebate is only received if there is an energy reduction. Otherwise, the user does not get any incentive or penalty (see Fig. 2.1). Mathematically,

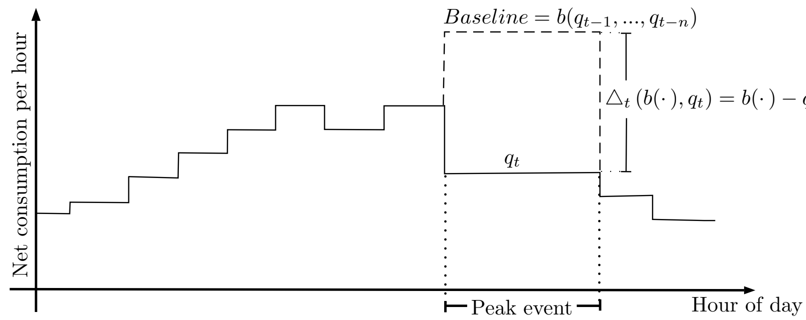

FIGURE 2.1: Baseline and rebate definition

Definition 8. *Let p_2 the rebate price received by the user due to energy reduction in peak periods. The PTR incentive π_2 is*

$$\pi_2\left(b\left(q_{t-1},...,q_{t-n}\right),q_t\right) = \begin{cases} p_2\left(\triangle_t\left(b(q_{t-1}...,q_{t-n}),q_t\right)\right) = p_2(b(q_{t-1},...,q_{t-n}) - q_t) & q_t < b \\ 0 & q_t \geq b \end{cases}$$

The consumer payoff function when he is enrolled in a PTR program is

$$U_t\left(q_t,\theta_t,b(q_{t-1},...,q_{t-n})\right) = G(q_t-\theta_t) - \pi(q_t) + r\pi_2(b(q_{t-1},...,q_{t-n}),q_t) \tag{2.4}$$

where r is a particular realization of a binary random variable R representing whether the consumer is called to participate in the program according to what the SO decides.

2.2 General problem formulation

The theories of Von-Neumann and Morgenstern are employed here to model decision-making under uncertainty. That is, the agent is assumed to behave as if he maximizes the expected value of the payoff function according to his actions and possible consequences. The reader can found more information in (Von Neumann and Morgenstern, 1944; Vega Redondo, 2003; Osborne, 1995). Subsequently, the consumer problem is to find the optimal decision when he is going to participate in a PTR program in order to increase his personal well-being and economic profit. In addition, the optimization formulae must include all the possible stochastic scenarios given the uncertainty of the variable θ.

The general problem formulation from the demand side is:

$$\max_{q_t,...,q_{t-n}\in[0,q_{max}]} E\left[U_{t-n}\left(q_{t-n},\theta_{t-n}\right) + ... + U_{t-1}\left(q_{t-1},\theta_{t-1}\right) + U_t\left(q_t,\theta_t,b(q_{t-1},...,q_{t-n})\right)\right] \tag{2.5}$$

where $E\left[\cdot\right]$ is the expectation operator. The optimization problem takes $n-1$ previous decisions to determine the best choice for all periods including the choice at the time t. Notice that the rebate price is only received at the period t (present time), namely, the payoff function at the time t is given by (2.4).

It is important to claim that the baseline could be estimated using several techniques according to the energy policies of each country or state. In (Mohajeryami, Doostan, and Schwarz, 2016) some methods for baseline calculation are found. The proposed solution for (2.5) is to formulate n-stages optimization problems solved backward in time. At stage i, the realization of θ_i is known. The stochastic programming algorithm,

1. $q_t^o\left(q_{t-1},...,q_{t-n};\theta_t\right) = \text{argmax}_{q_t\in[0,q_{max}]}\, U_t\left(q_t,\theta_t,b(q_{t-1},...,q_{t-n})\right)$
2. $q_{t-1}^o\left(q_{t-2},...,q_{t-n};\theta_{t-1}\right) = \text{argmax}_{q_{t-1}\in[0,q_{max}]}\, U_{t-1}\left(q_{t-1},\theta_{t-1}\right)$ $+E\left[U_t\left(q_t^o,\theta_t,b(q_{t-1},...,q_{t-n})\right)\right]$

$\vdots$

$n.$ $q_{t-n}^o(\theta_{t-n}) = \text{argmax}_{q_{t-n}\in[0,q_{max}]}\, U_{t-n}\left(q_{t-n},\theta_{t-n}\right) +$ $E\left[U_{t-(n-1)}\left(q_{t-n}^o,\theta_{t-n}\right) + ... + U_{t-1}\left(q_{t-1}^o,\theta_{t-1}\right) + U_t\left(q_t^o,\theta_t,b(q_{t-1}^o,q_{t-2}^o...,q_{t-n})\right)\right]$

It is vital to highlight that each period considered in this algorithm has similar features of consumption, i.e., consumer preferences and energy costs are the same in each period. For instance, the period between 7 and 8 pm for a week.

This Chapter focuses on the way to solve (2.5) for finding a closed form result for the consumer decision. In the next subsection, The stochastic optimization problem is solved for two periods. Furthermore, it is assumed that the user is always called to participate in PTR program, henceforth $r = 1$ is considered.

2.3 Problem formulation for two periods

A single previous period $t-1$ is assumed to estimate the baseline in eq. (2.3). Then, the baseline is $b(q_{t-1}) = q_{t-1}$. In this regard, the problem formulation is:

$$\max_{q_t,q_{t-1}\in[0,q_{max}]} E\left[U_{t-1}\left(q_{t-1},\theta_{t-1}\right) + U_t\left(q_t,\theta_t,b(q_{t-1})\right)\right] \tag{2.6}$$

First, the agent maximizes the energy consumption at the present time t, given that the realization of θ_t and the value of q_{t-1} are known.

$$q_t^o\left(q_{t-1};\theta_t\right) = \text{argmax}_{q_t\in[0,q_{max}]}\, U_t\left(q_t,\theta_t,b(q_{t-1})\right)$$

Second, the decision-maker determines the best consumption for the baseline setting period, knowing the rational choice q_t^o for the future. The realization of θ_{t-1} is given and the user faces uncertainty in θ_t only, i.e.,

$$q_{t-1}^o(\theta_{t-1}) = \text{argmax}_{q_{t-1}\in[0,q_{max}]}\, U_{t-1}\left(q_{t-1},\theta_{t-1}\right) + E\left[U_t\left(q_t^o,\theta_t,b(q_{t-1})\right)\right]$$

2.3.1 First-stage stochastic programming

The following result presents the solution q_t^o to the first-stage stochastic optimization at the time t.

Theorem 1. *The optimal consumption q_t^o of a user participating in a PTR program (i.e. the solution of the first-stage stochastic programming), given $G(\cdot)$ in (2.2) and $U_t(\cdot)$ in (2.4), is:*

$$q_t^o\left(q_{t-1};\theta_t\right)=\begin{cases} q_t^* & r=1 \text{ and } q_{t-1}-\overline{q}+\frac{p_2}{2\gamma}<\theta_t\leq\overline{\theta} & \textit{strategy A}\\ q_t^*-\frac{p_2}{\gamma} & r=1 \text{ and } \frac{p_2}{\gamma}-\overline{q}<\theta_t\leq q_{t-1}-\overline{q}+\frac{p_2}{2\gamma} & \textit{strategy B}\\ 0 & r=1 \text{ and } \underline{\theta}\leq\theta_t\leq\frac{p_2}{\gamma}-\overline{q} & \textit{strategy C}\\ q_t^* & r=0 & \textit{strategy D}\end{cases}$$

Note that when the SO or DR aggregator calls the user ($r=1$), strategy A means that the user decides rationally to spend q^* of energy (That is, he does not reduce power consumption), strategy B depicts whether the consumer chooses to diminish the demand to $q_t^*-\frac{p_2}{\gamma}$ and finally, strategy C is when the best decision is to consume zero energy.

According to Theorem 1, note that the best decision depends on the realization of θ_t. Then, the user chooses a strategy at the time t given his actual demand. The proof is shown in Subsection 2.6.1.

Collorary 1. *The expected value of the load q_t^o is:*

$$E\left[q_t^o\left(q_{t-1};\theta_t\right)\right]=\begin{cases} \overline{q} & r=1 \text{ and } p_2\leq 2\gamma\left(\overline{q}-q_{t-1}\right) & \textit{strategy A}\\ \overline{q}-\frac{p_2}{\gamma} & r=1 \text{ and } 2\gamma\left(\overline{q}-q_{t-1}\right)<p_2\leq\overline{q}\gamma & \textit{strategy B}\\ 0 & r=1 \text{ and } \overline{q}\gamma<p_2 & \textit{strategy C}\\ \overline{q} & r=0 & \textit{strategy D}\end{cases}$$

The expected value of consumer payoff $E\left[U_t\left(q_t^o,\theta_t,b(q_{t-1})\right)\right]$ is found assuming a continuous uniform distribution function $f_\Theta(\theta_t)$. Since $r=1$, from Theorem 1, the consumer has three available strategies according to the realization of uncertainty θ_t. In addition, the random variable is symmetric with respect to zero. Whether strategies A, B and C are feasible according to the parameters $\overline{\theta}$, $\underline{\theta}$, p_2, γ, $\overline{q}$ and the variable q_{t-1} then these strategies is within the probability density function of θ_t which it is shown in Fig. 2.2.

Looking in detail the intervals of θ_t that define strategy C, these depend of constant values, whereas the intervals for strategies A and B depend on the optimization variable q_{t-1}. Therefore, the probabilistic events change with q_{t-1}. For instance, strategy A has zero probability when $q_{t-1}>\overline{\theta}+\overline{q}-\frac{p_2}{2\gamma}$.

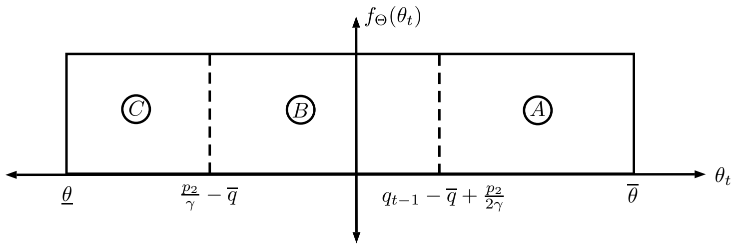

FIGURE 2.2: User optimal strategies within θ_t probability density function.

Collorary 2. *The expected value of the payoff in t, $E\left[U_t\left(q_t^o,\theta_t,b(q_{t-1})\right)\right]$, depends on the probabilities of available strategies. Therefore, a construction by cases is employed to solve $E\left[U_t\left(q_t^o,\theta_t,b(q_{t-1})\right)\right]$. There are three main cases:*

case 1: $\underline{\theta}>\frac{p_2}{\gamma}-\overline{q}$, strategy C does not exist. Then, $E\left[U_t\left(q_t^o,\theta_t,b(q_{t-1})\right)\right]$ depends on the value of q_{t-1}. Therefore,

$$E\left[U_t\left(q_t^o, \theta_t, b(q_{t-1})\right); \underline{\theta} > \frac{p_2}{\gamma} - \overline{q}\right] =$$
$$\begin{cases} E_A = \int_{\underline{\theta}}^{\overline{\theta}} U_t\left(q_t^*\right) f_\Theta(\theta_t) d\theta_t & q_{t-1} \in \left[0, \overline{q} + \underline{\theta} - \frac{p_2}{2\gamma}\right] \\ E_{AB} = \int_{q_{t-1}-\overline{q}+\frac{p_2}{2\gamma}}^{\overline{\theta}} U_t\left(q_t^*\right) f_\Theta(\theta_t) d\theta_t + & \\ \int_{\underline{\theta}}^{q_{t-1}-\overline{q}+\frac{p_2}{2\gamma}} U_t\left(q_t^* - \frac{p_2}{\gamma}\right) f_\Theta(\theta_t) d\theta_t & q_{t-1} \in \left[\overline{q} + \underline{\theta} - \frac{p_2}{2\gamma}, \overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}\right] \\ E_B = \int_{\underline{\theta}}^{\overline{\theta}} U_t\left(q_t^* - \frac{p_2}{\gamma}\right) f_\Theta(\theta_t) d\theta_t & q_{t-1} \in \left[\overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}, q_{max}\right] \end{cases}$$

case 2: $\underline{\theta} \leq \frac{p_2}{\gamma} - \overline{q} < \overline{\theta}$*, strategy C has positive probability. Therefore,* $E\left[U_t\left(q_t^o, \theta_t, b(q_{t-1})\right)\right]$ *is given by:*

$$E\left[U_t\left(q_t^o, \theta_t, b(q_{t-1})\right); \underline{\theta} \leq \frac{p_2}{\gamma} - \overline{q} < \overline{\theta}\right] =$$
$$\begin{cases} E_A = \int_{\underline{\theta}}^{\overline{\theta}} U_t\left(q_t^*\right) f_\Theta(\theta_t) d\theta_t & q_{t-1} \in \left[0, \overline{q} + \underline{\theta} - \frac{p_2}{2\gamma}\right] \\ E_{AC} = \int_{q_{t-1}-\overline{q}+\frac{p_2}{2\gamma}}^{\overline{\theta}} U_t\left(q_t^*\right) f_\Theta(\theta_t) d\theta_t + & \\ \int_{\underline{\theta}}^{q_{t-1}-\overline{q}+\frac{p_2}{2\gamma}} U_t\left(0\right) f_\Theta(\theta_t) d\theta_t & q_{t-1} \in \left[\overline{q} + \underline{\theta} - \frac{p_2}{2\gamma}, \frac{p_2}{2\gamma}\right] \\ E_{ABC} = \int_{q_{t-1}-\overline{q}+\frac{p_2}{2\gamma}}^{\overline{\theta}} U_t\left(q_t^*\right) f_\Theta(\theta_t) d\theta_t + & \\ \int_{\frac{p_2}{\gamma}-\overline{q}}^{q_{t-1}-\overline{q}+\frac{p_2}{2\gamma}} U_t\left(q_t^* - \frac{p_2}{\gamma}\right) f_\Theta(\theta_t) d\theta_t + & q_{t-1} \in \left[\frac{p_2}{2\gamma}, \overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}\right] \\ \int_{\underline{\theta}}^{\frac{p_2}{\gamma}-\overline{q}} U_t\left(0\right) f_\Theta(\theta_t) d\theta_t & \\ E_{BC} = \int_{\frac{p_2}{\gamma}-\overline{q}}^{\overline{\theta}} U_t\left(q_t^* - \frac{p_2}{\gamma}\right) f_\Theta(\theta_t) d\theta_t + & \\ \int_{\underline{\theta}}^{\frac{p_2}{\gamma}-\overline{q}} U_t\left(0\right) f_\Theta(\theta_t) d\theta_t & q_{t-1} \in \left[\overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}, q_{max}\right] \end{cases}$$

case 3: $\frac{p_2}{\gamma} - \overline{q} \geq \overline{\theta}$*, a priori, strategy C has probability one. However, the main point is* $q_{t-1} - \overline{q} + \frac{p_2}{\gamma}$ *then it could be exist other strategies different from C. Thus,* $E\left[U_t\left(q_t^o, \theta_t, b(q_{t-1})\right)\right]$ *is given by:*

$$E\left[U_t\left(q_t^o, \theta_t, b(q_{t-1})\right); \frac{p_2}{\gamma} - \overline{q} \geq \overline{\theta}\right] =$$
$$\begin{cases} E_A = \int_{\underline{\theta}}^{\overline{\theta}} U_t\left(q_t^*\right) f_\Theta(\theta_t) d\theta_t & q_{t-1} \in \left[0, \overline{q} + \underline{\theta} - \frac{p_2}{2\gamma}\right] \\ E_{AC'} = \int_{q_{t-1}-\overline{q}+\frac{p_2}{2\gamma}}^{\overline{\theta}} U_t\left(q_t^*\right) f_\Theta(\theta_t) d\theta_t + & \\ \int_{\underline{\theta}}^{q_{t-1}-\overline{q}+\frac{p_2}{2\gamma}} U_t\left(0\right) f_\Theta(\theta_t) d\theta_t & q_{t-1} \in \left[\overline{q} + \underline{\theta} - \frac{p_2}{2\gamma}, \overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}\right] \\ E_C == \int_{\underline{\theta}}^{\overline{\theta}} U_t\left(0\right) f_\Theta(\theta_t) d\theta_t & q_{t-1} \in \left[\overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}, q_{max}\right] \end{cases}$$

Note that the expected value $E\left[U_t\left(q_t^o, `_t, q_{t-1}\right)\right]$ is a piecewise function that depends on the value of q_{t-1}.

2.3.2 Second-stage stochastic programming

For the second-stage, the rational choice for q_t^o is known and the realization of θ_{t-1} is given. Then q_{t-1}^o is found by using the result of Theorem 1. The optimization problem is:

$$q_{t-1}^o(\theta_{t-1}) = \text{argmax}_{q_{t-1} \geq 0}\, G\left(q_{t-1} - \theta_{t-1}\right) - p q_{t-1} + E\left[G\left(q_t^o - \theta_t\right) - p q_t^o + r\pi_2\left(q_{t-1}, q_t^o\right)\right] \tag{2.7}$$

The mathematical solution of (2.7) is developed in the following three theorems for each case mentioned in the corollary 2 and the proofs are found in Subsections 2.6.2, 2.6.3 and 2.6.4.

Theorem 2. *Given* $\underline{\theta} > \frac{p_2}{\gamma} - \overline{q}$ *(case 1) and* $\overline{q} + \frac{p}{\gamma} > \overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}$*, then the optimal solution* q^o_{t-1} *for (2.7) is:*

$$E\left[q^o_{t-1}(\theta_{t-1})\right] = \begin{cases} \overline{q} - \frac{p_2}{2\gamma} + \frac{2p_2\overline{\theta}}{2\overline{\theta}\gamma - p_2} & 0 \leq p_2 < \frac{2}{3}\overline{\theta}\gamma \\ \overline{q} + \frac{p_2}{\gamma} & \frac{2}{3}\overline{\theta}\gamma \leq p_2 < p \\ q_{max} & p \leq p_2 < \gamma\left(\underline{\theta} + \overline{q}\right) \end{cases}$$

Theorem 3. *Given* $\underline{\theta} \leq \frac{p_2}{\gamma} - \overline{q} < \overline{\theta}$ *(case 2) and* $\overline{q} + \frac{p}{\gamma} > \overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}$*, then the optimal solution* q^o_{t-1} *for (2.7) is:*

$$E\left[q^o_{t-1}(\theta_{t-1})\right] = \begin{cases} \overline{q} - \frac{p_2}{2\gamma} + \frac{2p_2\overline{\theta}}{2\overline{\theta}\gamma - p_2} & \gamma\left(\underline{\theta} + \overline{q}\right) \leq p_2 < \frac{2}{3}\overline{\theta}\gamma \\ \overline{q} + \frac{p_2}{\gamma} & \frac{2}{3}\overline{\theta}\gamma \leq p_2 < p \\ q_{max} & p < p_2 \leq \gamma\left(\overline{\theta} + \overline{q}\right) \end{cases}$$

Theorem 4. *Given* $\frac{p_2}{\gamma} - \overline{q} \geq \overline{\theta}$ *(case 3) and* $\overline{q} + \frac{p}{\gamma} > \overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}$*, then the optimal solution* q^o_{t-1} *for (2.7) is:*

$$E\left[q^o_{t-1}(\theta_{t-1})\right] = \begin{cases} \overline{q} + \frac{p_2}{\gamma} & \gamma\left(\overline{\theta} + \overline{q}\right) \leq p_2 < p \\ q_{max} & p_2 > p \end{cases}$$

Theorems 2, 3 and 4 present the optimal consumption q_{t-1} given the solutions of Theorem 1. For Theorem 2, the result is rightful for incentives less than $\gamma\left(\underline{\theta} + \overline{q}\right)$, which means that strategy C does not exist. In addition, the saturation part of the consumer is $\overline{q} + \overline{\theta} - \frac{p_2}{2\gamma} < \overline{q} + \frac{p}{\gamma} < q_{max}$, namely, when $E\left[U_t\left(q^o_t, \theta_t, b(q_{t-1})\right); \underline{\theta} > \frac{p_2}{\gamma} - \overline{q}\right]$ is strategy B, specifically, $q_{t-1} \in \left[\overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}, q_{max}\right]$. Whether the user has low uncertainty, the Theorem 2 is employed for estimating optimal decision at the time $t-1$. Note that if $0 \leq p_2 < \frac{2}{3}\overline{\theta}\gamma$ the solutions is decreasing with respect to p_2, therefore, the situation when the incentive is too small, it is risky to increase the energy consumption at the baseline setting period. Nonetheless, this event is not common owing to the incentive is equal or greater than retail price. Next, whether $\frac{2}{3}\overline{\theta}\gamma \leq p_2 < p$ then the optimal strategies is to increase $\overline{q} + \frac{p_2}{\gamma}$. Finally, if the incentive is greater than p then the optimal choice is to increase the energy consumption as much as possible. Moreover, The meaning of Theorem 2 is the same than the Theorem 3. However, the consumer uncertainty is larger and the incentive limit is given by $\gamma\left(\underline{\theta} + \overline{q}\right) < p_2 < \gamma\left(\overline{\theta} + \overline{q}\right)$. Finally, Theorem 4 is valid for $p_2 \geq \gamma\left(\overline{\theta} + \overline{q}\right)$ and $p > \gamma\overline{\theta} - \frac{p_2}{2}$. Note that there are only two solutions that depend on incentive p_2. The uncertainty is greater than the previous two Theorems. In general, The saturation of consumer preferences causes that the user wastes energy.

2.4 Numerical examples

In this subsection, simulation results are presented to illustrate the optimal behavior of a user when he is participating in a PTR program. The utility function for this example is

$$G\left(q_t-\theta_t\right)=\begin{cases} -\frac{\gamma}{2}\left(q_t-q_t^*\right)^2+p\left(q_t-q_t^*\right)+\frac{\gamma}{2}\overline{q}^2+p\overline{q} & 0\leq q_t\leq \overline{q}+\frac{p}{\gamma}+\theta_t \\ p\overline{q}+\frac{p^2}{2\gamma}+\frac{\gamma\overline{q}^2}{2} & q_t>\overline{q}+\frac{p}{\gamma}+\theta_t \end{cases}$$

The retail price is $p = \$0.26/kWh$ (based on peak summer rate in 10/1/16 by Pacific Gas and Electric Company in San Francisco, California), deterministic baseline $\overline{q} = 8kWh$ and the curvature $\gamma = 0.05$. Randomness θ_t for each period has been created as a uniform random variable with zero mean and with symmetric support. A Monte Carlo simulation is performed with 10000 realizations of θ_t for each value of q_{t-1}.

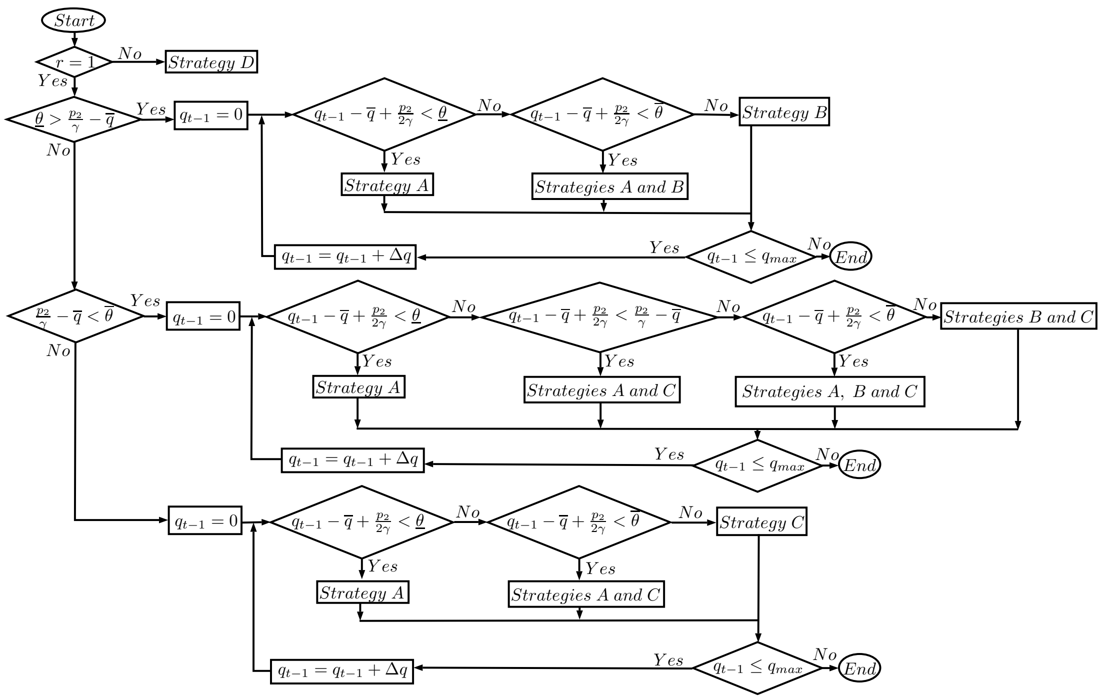

FIGURE 2.3: Flowchart to calculate the expected value of $U_t(\cdot)$

The flowchart in Fig. 2.3. shows the conditions to determine the case that the user faces for choosing his decision according to the value of q_{t-1}, in order to solve the expected value in (2.7). This flowchart is derived from Fig. 2.2 and corollary 2 by analyzing when strategies have positive probability.

2.4.1 Incentives analysis

In this subsection, the effect of the incentive p_2 on the load and user utility at time t given the baseline q_{t-1} is studied by changing the reward p_2. For this analysis, $\theta_t \sim$ unif $[-0.25\overline{q}, 0.25\overline{q}]$ and $q_{max} = 20kWh$ are assumed. In Fig. 2.4 are shown three different situations that depend on the incentive value. The first column, the reward p_2 is presented for each situation. The second one, the plot of energy consumptions at the time t versus the consumption at the time $t-1$ are shown according to the incentive. Third column, the expected value of profit function based on the decisions at time $t-1$.

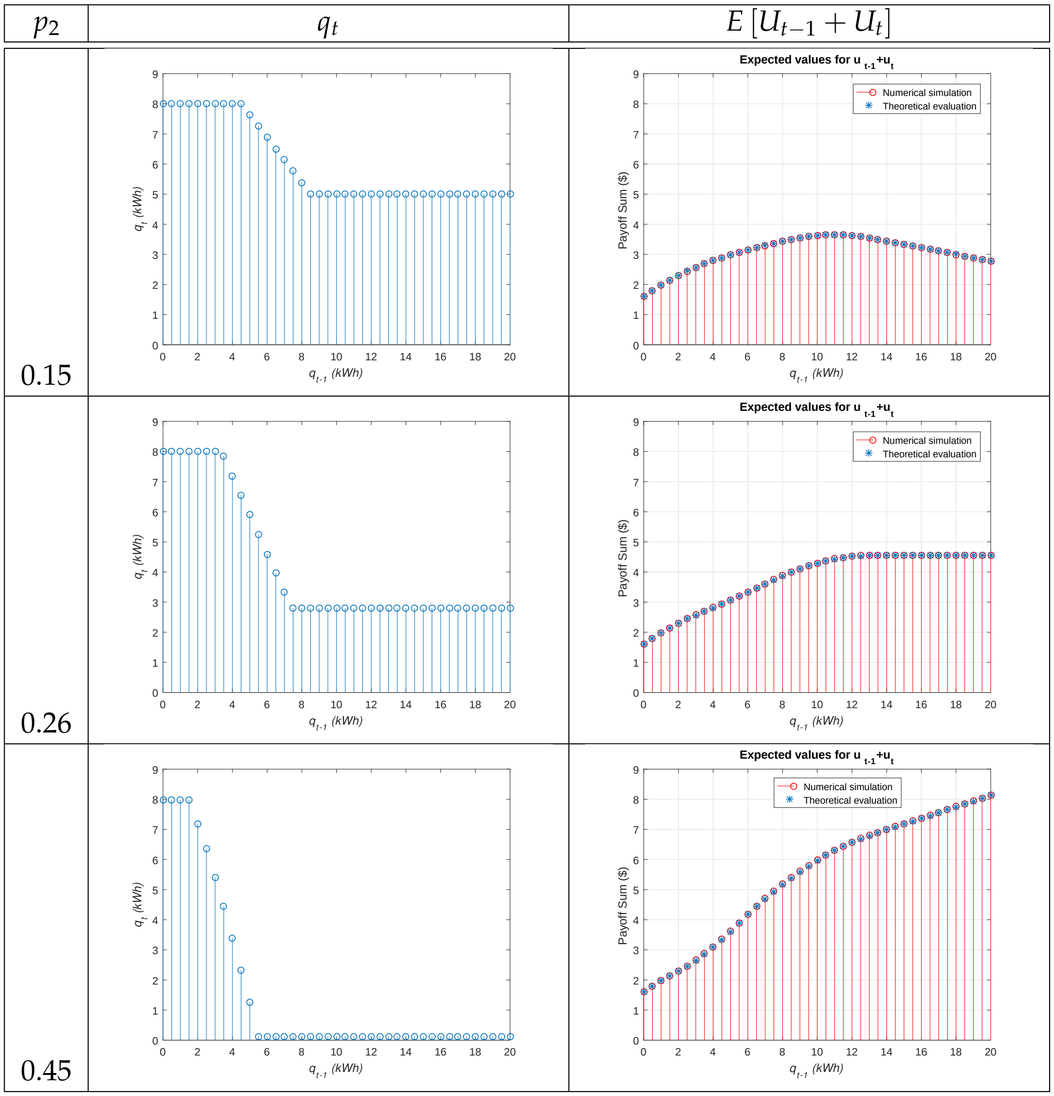

FIGURE 2.4: Incentives analysis.

First, the event when the incentive is lower than retail price, i.e., $p_2 < p$ is evaluated. For $p_2 = \$0.15/kWh$, the optimal solution is to increase energy consumption at the period $t-1$, close to $q_{t-1} = \overline{q} + \theta_{t-1} + \frac{p_2}{\gamma} = 11kWh$ and reduce energy consumption at t to $q_t^o = \overline{q} + \theta_t - \frac{p_2}{\gamma} = 5kWh$. Next, when the incentive and the retail price are the same $p_2 = p$. The rational user consumes at the past time whatever value comprising in $q_{t-1} = [11kWh, 20kWh]$, then, taking into account the worst event, the user consumes $q_{t-1} = 20kWh$, therefore, the optimal consumption at the time t, it is $q_t^o = \overline{q} + \theta_t - \frac{p_2}{\gamma} = 2.79kWh$. Finally, the situation when the incentive is greater than retail price, namely, $p_2 > p$ is assessed. For $p_2 = \$0.45/kWh$, the optimal behavior is to consume as much energy as possible, $q_{t-1} = q_{max}$, irrespective of parameters $\gamma, \overline{q}, \overline{\theta}$ and $\underline{\theta}$. If the maximum value is $q_{t-1} = 20kWh$ then, he would consume zero energy $q_t = 0kWh$ at the time t in order to get the maximum profit. These behaviors are predicted by corollary 1 and Theorem 2.

p_2 (\$/kWh)	Expected profit (\$)	q^o_{t-1} (kWh)	q^o_t (kWh)	$q^o_{t-1} + q^o_t$ (kWh)
0	3.2	8	8	16
0.15	3.65	11	5	16
0.26	4.55	20	2.79	22.79
0.45	8.13	20	0	20

TABLE 2.1: Comparison of optimal user strategies under different incentives.

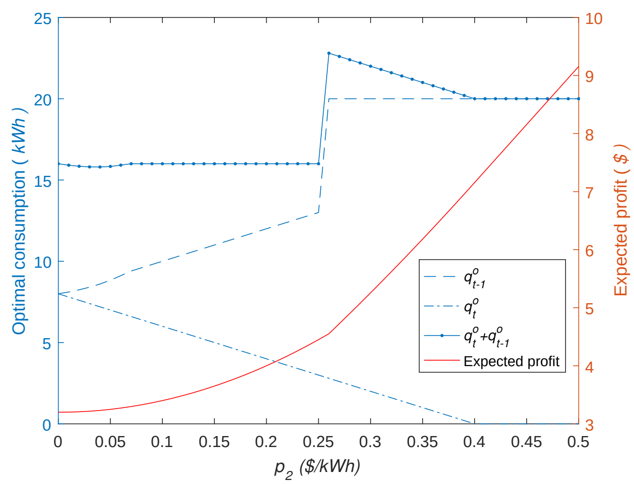

FIGURE 2.5: Optimal consumption and profit with 25% of uncertainty.

The previous results are summarized in table 2.1. Whether the user is not called or the incentive is zero then the trivial solution is not to alter his behavior. On the other hand, when the price incentive is higher than zero but lower than the retail price ($p_2 = 0.15$), the user is induced to raise his consumption to alter the baseline and get the highest economic benefits by reducing the consumption at time t, getting a profit of \$3.65. Likewise, whether the agent gets an incentive equal or greater than the retail price then he alters his consumption up to the maximum possible load to maximize the profit to \$4.55 or \$8.13 according to the incentive, consuming much more energy than in the previous situations.This alteration of the baseline causes economic inefficiency to the SO or DR aggregator. A mechanism design should be designed in order to manage properly the signal r to face this problem when the DR program is based on baseline method.

Lastly, in Fig. 2.5 are shown the optimal consumption at the time $t-1$ and t, the net consumption ($q_t + q_{t-1}$) and the expected value of consumer profit versus the incentive payment p_2. The optimal decision at the setting time is to increase the consumption as the incentive is raising, namely, the user alters the baseline in order to improve his profit. Note that whether $p_2 > \$0.26/kWh$ the energy expenditure is saturated to $q_{max} = 20kWh$. In addition, the rational choice at the period t is to diminish the energy consumption to receive the benefits of participating in the PTR program. For $p_2 > \$0.4/kWh$, the consumed energy goes to zero. Besides, whether $p_2 \in [0, 0.26)$ $\$/kWh$, the net consumption is less or equal to $16kWh$, that is, the user shifts his energy consumption. On the other hand, for $p_2 > \$0.26/kWh$, the user spends more energy that he needs, taking into account all periods. Finally, the expected value of consumer profit is an increasing function, thus, the incentive payment improve the consumer benefits. However, the PTR contract is favorable for the SO or DR aggregator as long as $p_2 < p$ because the consumer is shifting his energy consumption. In other situations, the incentive goes against with the objectives of a DR program.

2.4.2 Uncertainty variation

In this part, user behaviors for different uncertainty levels are analyzed. The realization of uniform random variable θ_t is settled with four different supports in order to assess the uncertainty. These supports are proposed as percentages of the deterministic baseline $\bar{q}$. For this survey are considered the following percentages: 10%, 30%, 50% and 90%. In fig 2.6 is compared the optimal decision q_t for all the stated uncertainties. Note that the optimal choice at time t does not depend on the uncertainty level owing to in this period. Moreover, in fig 2.8 is shown the rational choices at the period $t-1$. For $p_2 \in [0, 0.26)$, the user with high uncertainty (e.g. with 50%) should spend less energy than a predictable consumer (e.g. with 10%) since his consumption is unknown then he reduces his consumption for facing this variation and pursuing the benefits of the PTR program.Whether the incentive is greater than the retail price, hence, all decisions are saturated. Furthermore, A similar behavior is found whether the net consumption is analyzed (see Fig. 2.7). It is vital to restate that the consumption variation is perceived when the incentive is lower than the retail price. Lastly, In Fig. 2.9 is presented the expected value of consumer profit. The expected profits are the same for all percentages because each situation has the same preferences. In brief, the uncertainty affects low payments of incentive, therefore, the user does not have certainty related to his consumption pattern under this conditions, then, his best strategy is to be cautious and spend less energy than a predictable consumer.

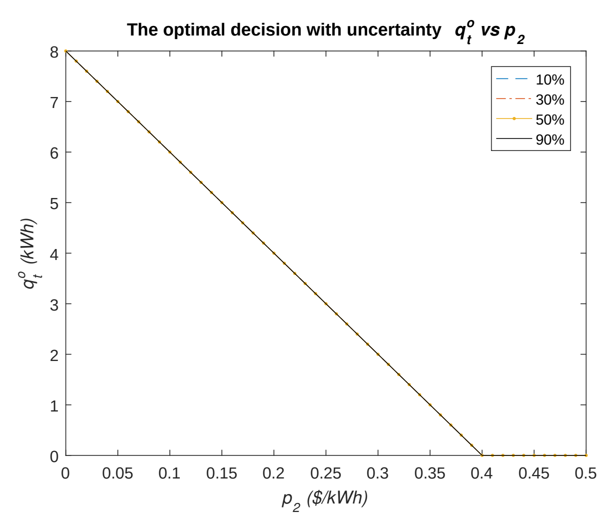

FIGURE 2.6: Optimal decision for q_t with uncertainty of 10%, 30%, 50% and 90% according to the incentive.

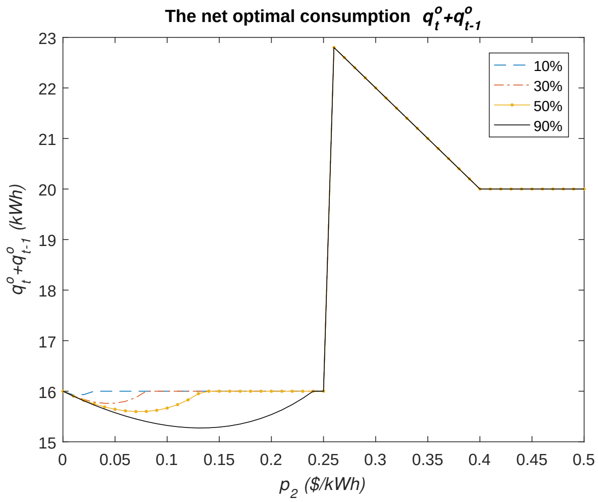

FIGURE 2.7: Net consumption with uncertainty of 10%, 30%, 50% and 90% according to the incentive.

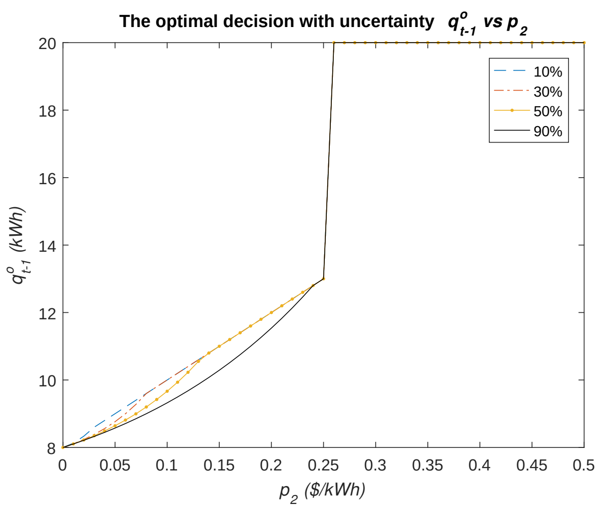

FIGURE 2.8: Optimal decision for q_{t-1} with uncertainty of 10%, 30%, 50% and 90% according to the incentive.

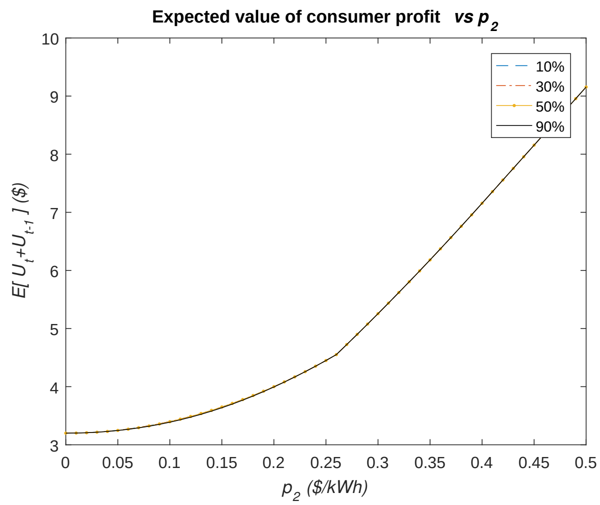

FIGURE 2.9: Expected value for $U_{t-1} + U_t$ with uncertainty of 10%, 30%, 50% and 90% according to the incentive.

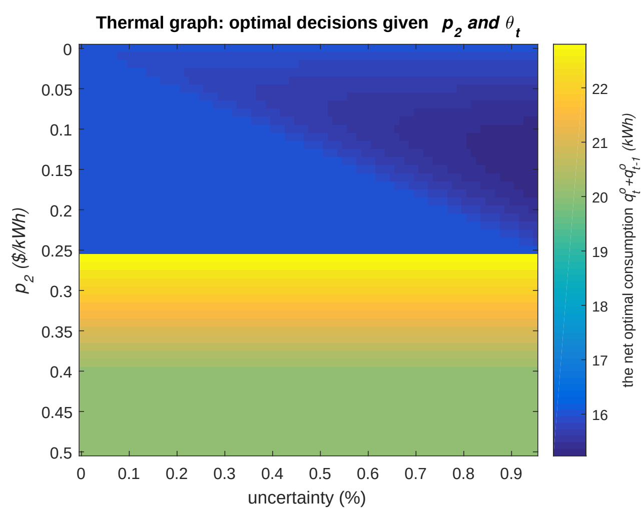

FIGURE 2.10: Thermal graph of optimal decisions given the incentive and uncertainty variation.

Finally, in fig 2.10 is shown a thermal graph of the net optimal consumption according to the incentive price p_2 and uncertainty variation θ_t as a plot summary. An important threshold is when the incentive is equal to the retail price, i.e., $p_2 = \$0.26/kWh$. Even more, the maximum consumption is detected when p_2 is just slightly higher than p_2, rising around 22 kWh represented by a yellow color. In this situation, the optimal consumption does not change with the uncertainty level. In addition, the rational consumption decreases for incentives between $\$0.26/kWh$ to $\$0.4/kWh$. For higher incentives, the net consumption remains constant in $20kWh$. On the other hand, when the incentive is lower than the retail price, the optimal consumption depends on uncertainty variation. If a user is not sure of his demand then the optimal choice is to consume less energy than a predictable consumer. In particular, this non-linear pattern is depicted by variations in blue tones of Fig. 2.10. Furthermore, the maximum energy consumption is 16 kWh for p_2 lower than p, therefore, a rational consumer shifts or reduces his load requirement under this incentive conditions.

2.5 Findings

In this Chapter was analyzed the rational behavior of a consumer that participates in a PTR program within an electricity market. The problem was addressed using a stochastic programming algorithm. A closed-form solution was found for a two-periods framework. The previous consumption was taken as the baseline and it was assumed that the user is always called to participate in the PTR program. The formulation allowed linking the consumer decisions among different consumption periods. Furthermore, uncertainty in load requirements was considered and coupled through conditional expectation.

It was found that a rational user changes his consumption pattern in order to alter the baseline construction and increase his well-being. Whether the incentive is lower than the regular energy price, the user's best strategy is to shift the energy consumption from the DR event to the baseline settling period. Otherwise, whether the incentive is greater than the retail price then the consumer maximizes his profits consuming as much energy as possible during the baseline setting period, harming the system reliability. In addition, the effect of uncertainty in the consumer energy requirement was analyzed. It was found that the best decision for a consumer with high uncertainty is to spend less energy than a predictable user.

PTR programs aim to induce users to reduce their energy consumption during a peak event. However, the analysis of the proposed model showed that in most cases, users shift or increase their demand in order to maximize their profits. Only those consumers with high levels of uncertainty reduce their consumption when the incentive is lower than the retail price. Therefore, it was found that a PTR program is not suitable if the SO is seeking a net reduction of energy consumption on the demand side.

2.6 Mathematical proofs

2.6.1 Proof of the Theorem 1

Proof. The optimization problem is analyzed by intervals according to the established setting. Then the global maximum is found.

Strategy A1: $r = 1$ (Called), $q_t^o \geq q_{t-1}$ (Non-participant) and $0 \leq_t^o \leq q_t^* + \frac{p}{\gamma}$ (G non-saturated)

$$[q_t^o] = \text{argmax}_{q_t \in \left[0, q_t^* + \frac{p}{\gamma}\right]} \; -\frac{\gamma}{2}\left(q_t - q_t^*\right)^2 + p\left(q_t - q_t^*\right) + k - pq_t$$

The first-order optimality condition yields to

$$q_t^0 = q_t^* = \overline{q} + \theta_t \tag{2.8}$$

Strategy A2: $r = 1$ (Called), $q_t^o \geq q_{t-1}$(Non-participant) and $q_t^o > q_t^* + \frac{p}{\gamma}$ (G saturated).

$$[q_t^o] = \text{argmax}_{q_t \in \left[q_t^* + \frac{p}{\gamma}, q_{max}\right]} \; -\frac{p^2}{2\gamma} + \frac{p^2}{\gamma} + k - pq_t$$

This function is unbounded below. The corner solution is

$$q_t^o = q_t^* + \frac{p}{\gamma} \tag{2.9}$$

Comparing the optimal solutions (2.8) and (2.9), the optimal strategy is (2.8) when the user is called but does not participate in DR.

Strategy B1: $r = 1$ (Called), $q_t^o < q_{t-1}$ (Participant), and $0 \leq q_t^o \leq q_t^* + \frac{p}{\gamma}$ (G non-saturated).

$$[q_t^o] = \text{argmax}_{q_t \in \left[0, q_t^* + \frac{p}{\gamma}\right]} \; -\frac{\gamma}{2}\left(q_t - q_t^*\right)^2 + p\left(q_t - q_t^*\right) + k - pq_t + p_2\left(q_{t-1} - q_t\right)$$

The first-order optimality condition yields to

$$q_t^0 = q_t^* - \frac{p_2}{\gamma} = \bar{q} + \theta_t - \frac{p_2}{\gamma} \tag{2.10}$$

Strategy B2: $r = 1$ (Called), $q_t^o < q_{t-1}$ (Participant) and $q_t^o > q_t^* + \frac{p}{\gamma}$ (Saturated).

$$[q_t^o] = \text{argmax}_{q_t \in \left[q_t^* + \frac{p}{\gamma}, q_{max}\right]} \; -\frac{p^2}{2\gamma} + \frac{p^2}{\gamma} + k - pq_t + p_2\left(q_{t-1} - q_t\right)$$

This function is unbounded below. The corner solution is

$$q_t^o = q_t^* + \frac{p}{\gamma} \tag{2.11}$$

Comparing the optimal solutions (2.10) and (2.11), the optimal strategy is (2.10) when the user is called and participates in DR.

Strategy C: Importantly, the incentive p_2 can be so high to drive (2.10) negative values. As there is no sense in a negative consumption, the problem is limited to $[0, q_{max}]$, then

$$q_t^o = 0 \;\text{ if }\; \theta_t \leq \frac{p_2}{\gamma} - \bar{q} \tag{2.12}$$

Strategy D1: $r = 0$ (Non-called) and $0 \leq q_t^o \leq q_t^* + \frac{p}{\gamma}$ (Non-saturated)

$$[q_t^o] = \text{argmax}_{q_t \in \left[0, q_t^* + \frac{p}{\gamma}\right]} \; -\frac{\gamma}{2}\left(q_t - q_t^*\right)^2 + p\left(q_t - q_t^*\right) + k - pq_t$$

The first-order optimality condition yields to

$$q_t^0 = q_t^* = \bar{q} + \theta_t \tag{2.13}$$

Strategy D2: $r = 0$ (Non-called) and $q_t^o > q_t^* + \frac{p}{\gamma}$ (saturated)

$$[q_t^o] = \text{argmax}_{q_t \in \left[q_t^* + \frac{p}{\gamma}, q_{max}\right]} \; -\frac{p^2}{2\gamma} + \frac{p^2}{\gamma} + k - pq_t$$

This function is unbounded below. The corner solution is

$$q_t^o = q_t^* + \frac{p}{\gamma} \tag{2.14}$$

Comparing (2.13) and (2.14), the optimal strategy when the user is not called is (2.13).

Note that, when called ($r = 1$), the user decides to participate (*Strategy B*) when $q_t^o < q_{t-1}$, i.e., $\theta_t < q_{t-1} - \overline{q} + \frac{p_2}{\gamma}$. While the user does not participate (*Strategy A*) when $q_t^o > q_{t-1}$, i.e., $\theta_t > q_{t-1} - \overline{q}$. Then, for any realization of the additive uncertainty θ_t within the interval $q_{t-1} - \overline{q} < \theta_t < q_{t-1} - \overline{q} + \frac{p_2}{\gamma}$, there are two local maxima.

In order to find the global solution, the payoff in strategies A and B are compared. The critical value of θ_t that provides the same payoff in both strategies is:

$$U\left(\overline{q} + \theta_t - \frac{p_2}{\gamma}, \theta_t, q_{t-1}\right) = U\left(\overline{q} + \theta_t, \theta_t, q_{t-1}\right) \tag{2.15}$$

Solving for θ_t,

$$\theta_t = q_{t-1} - \overline{q} + \frac{p_2}{2\gamma} \tag{2.16}$$

Eq. (2.16) gives the limit of the uncertain load when the user commutes from strategy A to strategy B.

Organizing by intervals the results (2.13), (2.8), (2.10) and (2.12), the solution is given by Theorem 1. □

2.6.2 Proof of the Theorem 2

Proof. Let $r = 1$, i.e., the user is always called to participate in the PTR program. Under the assumption (see the corollary 2 or the Fig. 2.3) that $\underline{\theta} > \frac{p_2}{\gamma} - \overline{q}$, namely, strategy C does not exist in the density probability function $f_\theta(\theta_t)$ (See Fig. 2.2 with zero probability for strategy C). Also, it is assumed that $\overline{\theta} = -\underline{\theta}$ and $\underline{\theta} > \frac{p_2}{\gamma} - \overline{q}$ and the parameters $\overline{q}$ and p_2 are positives.

Subsequently, the net payoff function for all periods is given by figure 2.11. Notice that the intervals are given when the conditions $q_{t-1} - \overline{q} + \frac{p_2}{2\gamma}$ is equal to $\underline{\theta}$ and $\overline{\theta}$. Besides, the saturation part according the utility function (equation (2.2)) is assumed between $\overline{q} + \overline{\theta} - \frac{p_2}{2\gamma} \leq \overline{q} + \frac{p}{\gamma} \leq q_{max}$. This assumption about the saturation part is motivated due to $\overline{\theta}$ is relatively small when the user has not too much uncertainty. Thus, an optimization problem is formulated by intervals according to strategies that are feasible. Then, the maximum global is found comparing all the local maxima.

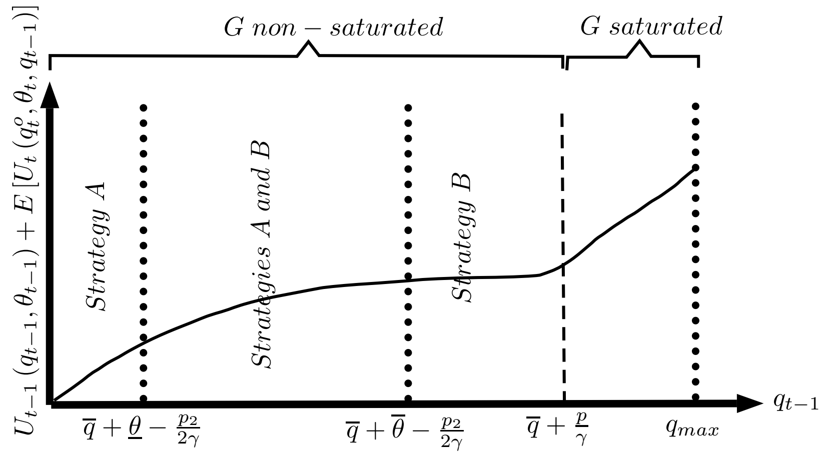

FIGURE 2.11: Net payoff function when strategy C does not exist.

The first local maximum is found when strategy A is feasible.

$$\left[q_{t-1}^o\right] = \text{argmax}_{q_{t-1}}\ G\left(q_{t-1}-\theta_{t-1}\right) - pq_{t-1} + E_A \\ \text{s.t.}\quad 0 \leq q_{t-1} \leq \overline{q} + \underline{\theta} - \tfrac{p_2}{2\gamma}$$

The Karush Kuhn Tucker conditions for the above formulations are:

$$\tfrac{\partial}{\partial q_{t-1}}\left(G\left(q_{t-1}-\theta_{t-1}\right) - pq_{t-1} + E_A\right) + \mu_1 - \mu_2 = 0 \\ 0 \leq q_{t-1} \perp \mu_1 \geq 0 \\ q_{t-1} \leq \overline{q} + \underline{\theta} - \tfrac{p_2}{2\gamma} \perp \mu_2 \geq 0$$

Being the $E\left[\theta_{t-1}\right] = 0$, it is found that:

$$E\left[q_{t-1}^o(\theta_{t-1}) \mid \underline{\theta} > \frac{p_2}{\gamma} - \overline{q}\, and\, q_{t-1} \in \left[0, \overline{q} + \underline{\theta} - \frac{p_2}{2\gamma}\right]\right] = \begin{cases} 0 & -\overline{q} > 0 \\ \overline{q} & \overline{q} \geq 0\, and\, p_2 < 2\gamma\underline{\theta} \\ \overline{q} + \underline{\theta} - \frac{p_2}{2\gamma} & p_2 \geq 2\gamma\underline{\theta} \end{cases}$$

Therefore the unique feasible solution for this situation is:

$$E\left[q_{t-1}^o(\theta_{t-1}) \mid \underline{\theta} > \frac{p_2}{\gamma} - \overline{q}\, and\, q_{t-1} \in \left[0, \overline{q} + \underline{\theta} - \frac{p_2}{2\gamma}\right]\right] = \overline{q} + \underline{\theta} - \frac{p_2}{2\gamma} \qquad p_2 \geq 0 \tag{2.17}$$

In addition, The sufficient condition is guaranteed, i.e., $-\gamma < 0$

Next, local maxima when strategies A and B are feasible is found solving the following optimization problem:

$$\left[q_{t-1}^o\right] = \text{argmax}_{q_{t-1}}\ G\left(q_{t-1}-\theta_{t-1}\right) - pq_{t-1} + E_{AB} \\ \text{s.t.}\quad \overline{q} + \underline{\theta} - \tfrac{p_2}{2\gamma} \leq q_{t-1} \leq \overline{q} + \overline{\theta} - \tfrac{p_2}{2\gamma}$$

A similar analysis using KKT conditions yields the following result:

$$E\left[q_{t-1}^o(\theta_{t-1}) \mid \underline{\theta} > \frac{p_2}{\gamma} - \overline{q}\, and\, q_{t-1} \in \left[\overline{q} + \underline{\theta} - \frac{p_2}{2\gamma}, \overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}\right]\right] =$$

$$\begin{cases} \overline{q}+\underline{\theta}-\frac{p_2}{2\gamma} & p_2<2\underline{\theta}\gamma \\ \overline{q}-\frac{p_2}{2\gamma}+\frac{p_2\left(\overline{\theta}-3\underline{\theta}\right)}{2\left(\left(\overline{\theta}-\underline{\theta}\right)\gamma-p_2\right)} & 2\underline{\theta}\gamma\leq p_2<\frac{2}{3}\overline{\theta}\gamma \\ \overline{q}+\overline{\theta}-\frac{p_2}{2\gamma} & p_2\geq\frac{2}{3}\overline{\theta}\gamma \end{cases}$$

Then as well, p_2 is positive, resulting

$$E\left[q^o_{t-1}(\theta_{t-1})\mid\underline{\theta}>\frac{p_2}{\gamma}-\overline{q}\, and\, q_{t-1}\in\left[\overline{q}+\underline{\theta}-\frac{p_2}{2\gamma},\overline{q}+\overline{\theta}-\frac{p_2}{2\gamma}\right]\right]= \begin{cases} \overline{q}-\frac{p_2}{2\gamma}+\frac{p_2\left(\overline{\theta}-3\underline{\theta}\right)}{2\left(\left(\overline{\theta}-\underline{\theta}\right)\gamma-p_2\right)} & 0\leq p_2<\frac{2}{3}\overline{\theta}\gamma \\ \overline{q}+\overline{\theta}-\frac{p_2}{2\gamma} & p_2\geq\frac{2}{3}\overline{\theta}\gamma \end{cases} \tag{2.18}$$

However, the sufficient condition is met when $p_2<\gamma\left(\overline{\theta}-\underline{\theta}\right)$. In other circumstances, the solution will be a corner. Given that $\gamma\left(\overline{\theta}-\underline{\theta}\right)>\frac{2}{3}\overline{\theta}\gamma$ then the solution is the same.

Finally, the local maximum when strategy B is feasible.

$$\begin{aligned}\left[q^o_{t-1}\right] &= \text{argmax}_{q_{t-1}}\ G\left(q_{t-1}-\theta_{t-1}\right)-pq_{t-1}+E_B \\ &\text{s.t.}\quad \overline{q}+\overline{\theta}-\frac{p_2}{2\gamma}\leq q_{t-1}\leq q_{max}\end{aligned}$$

which has the following solution,

$$E\left[q^o_{t-1}(\theta_{t-1})\mid\underline{\theta}>\frac{p_2}{\gamma}-\overline{q}\, and\, q_{t-1}\in\left[\overline{q}+\overline{\theta}-\frac{p_2}{2\gamma},q_{max}\right]\right]=\begin{cases} \overline{q}+\overline{\theta}-\frac{p_2}{2\gamma} & 0\leq p_2<\frac{2}{3}\overline{\theta}\gamma \\ \overline{q}+\frac{p_2}{\gamma} & \frac{2}{3}\overline{\theta}\gamma\leq p_2<p \\ q_{max} & p_2\geq p \end{cases} \tag{2.19}$$

Furthermore, The minimum condition is guaranteed, i.e., $-\gamma<0$.

Lastly, comparing the net payoff at the local maxima given by (2.17), (2.18) and (2.19). The global solution is:

$$E\left[q^o_{t-1}(\theta_{t-1})\mid\underline{\theta}>\frac{p_2}{\gamma}-\overline{q}\right]=\begin{cases} \overline{q}-\frac{p_2}{2\gamma}+\frac{2p_2\overline{\theta}}{2\overline{\theta}\gamma-p_2} & 0\leq p_2<\frac{2}{3}\overline{\theta}\gamma \\ \overline{q}+\frac{p_2}{\gamma} & \frac{2}{3}\overline{\theta}\gamma\leq p_2<p \\ q_{max} & p\leq p_2<\gamma\left(\underline{\theta}+\overline{q}\right) \end{cases}$$

□

2.6.3 Proof of the Theorem 3

Proof. The mathematical expression $\frac{p_2}{\gamma}-\overline{q}$ is located within the limits of the probability density function (see Fig. 2.2). Furthermore, $\overline{q}+\frac{p}{\gamma}>\overline{q}+\frac{p}{\gamma}$, i.e., the saturated point is when strategies B and C are feasible as it is shown in Fig. 2.12. Also, let $\underline{\theta}\leq\frac{p_2}{\gamma}-\overline{q}<\overline{\theta}$.

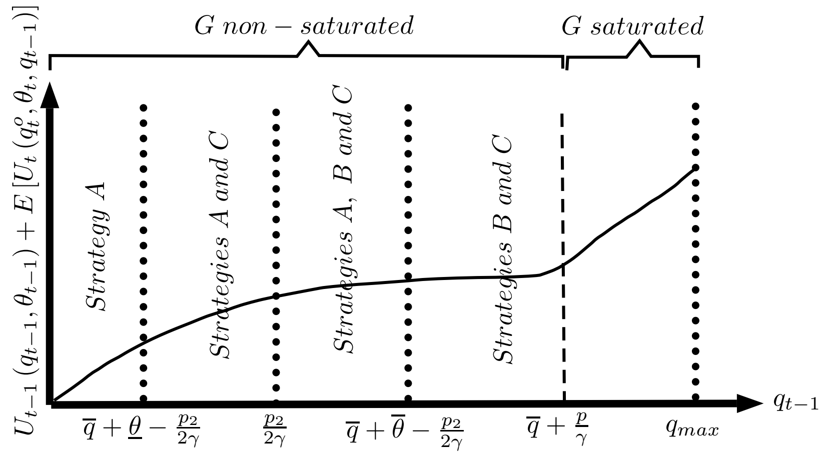

FIGURE 2.12: Net payoff function when strategy C is inside of probability density function.

It is uncomplicated to show the following statements

$$\max_{q_{t-1}} G(q_{t-1} - \theta_{t-1}) - pq_{t-1} + E_{AB} = \max_{q_{t-1}} G(q_{t-1} - \theta_{t-1}) - pq_{t-1} + E_{ABC}$$

$$\max_{q_{t-1}} G(q_{t-1} - \theta_{t-1}) - pq_{t-1} + E_{B} = \max_{q_{t-1}} G(q_{t-1} - \theta_{t-1}) - pq_{t-1} + E_{BC}$$

Therefore, the last two zones ($\left[q_{max}, \overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}\right]$ and $\left[\overline{q} + \overline{\theta} - \frac{p_2}{2\gamma}, \frac{p_2}{2\gamma}\right]$) from Fig.2.11 and Fig. 2.12 have some similarities. Whether the reader follows the same steps of the proof of the Theorem 2 then the solution for this Theorem is:

$$E\left[q_{t-1}^o(\theta_{t-1}) \mid \underline{\theta} < \frac{p_2}{\gamma} - \overline{q} < \overline{\theta}\right] = \begin{cases} \overline{q} - \frac{p_2}{2\gamma} + \frac{2p_2\overline{\theta}}{2\overline{\theta}\gamma - p_2} & \gamma(\underline{\theta} + \overline{q}) \leq p_2 < \frac{2}{3}\overline{\theta}\gamma \\ \overline{q} + \frac{p_2}{\gamma} & \frac{2}{3}\overline{\theta}\gamma \leq p_2 < p \\ q_{max} & p < p_2 \leq \gamma(\overline{\theta} + \overline{q}) \end{cases}$$

□

2.6.4 Proof of the Theorem 4

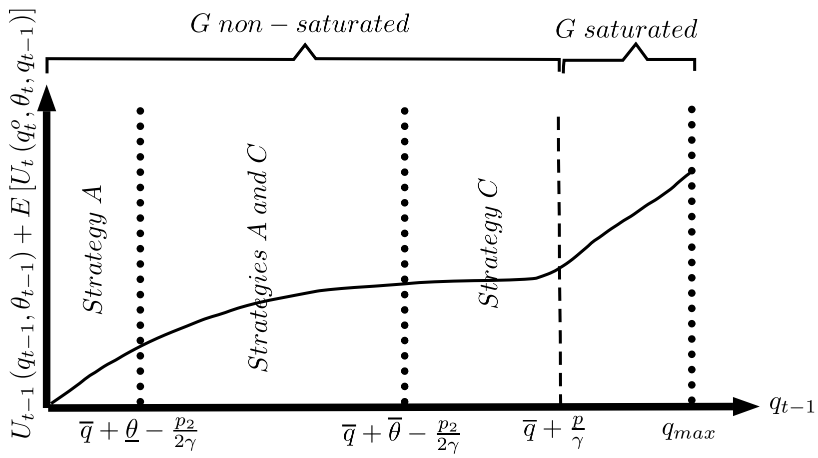

FIGURE 2.13: Net payoff function when strategy C is greater than $\overline{\theta}$.

This Theorem is proved using the same procedure than Theorem (2) and (3). Fig. 2.13 depicts all the zones feasible for this case.

Chapter 3

A contract for demand response based on probability of call

A novel contract is proposed by addressing the gaming problem quantified in the Chapter 2 and described in Subsection 1.1.1, through a new concept called the probability of call to limit the baseline alteration of each participant consumer. Literature review about mechanism designs and contracts in DR was presented in Subsection 1.1.2, showing some concerns from an implementation viewpoint. The probability of call can be understood as the chance of agent to be chosen by the aggregator to serve as DR resource during peak times. In this contract, a user submits his baseline and reduction capacity. This approach does not require marginal utility information as in traditional mechanisms, which could be a private parameter difficult to estimate by a consumer. Accordingly, agents bid two quantities in terms of energy, then this contract is a more intuitive procedure and can be suitable to be implemented in practice. Therefore, the main goal of the aggregator is to select randomly which participant agents are called to performed DR in order to manage the truth-telling behavior of each agent through the probability of call. The results of this Chapter were derived in the publications (Vuelvas, Ruiz, and Gruosso, 2018a; Vuelvas, Ruiz, and Gruosso, 2018b). The key points are summarized as follows:

- The optimal decision problem is presented by 2-stage contract. The result is obtained backward in time to find the optimal choice that user faces at each time. Theoretical analysis and numerical studies are provided to demonstrate the benefits and properties. The outcome shows that the contract is individually rational (voluntary participation), incentive compatibility (truth-telling) on the reported reduction capacity and asymptotically incentive compatibility on the reported baseline.

- In this contract, the aggregator does no require to estimate/forecast the customer baseline, then it only calls randomly participant consumers with a probability of call close to zero in order to obtain truthful behavior by demand side. In addition, There is no need for agents to inform their full types to achieve good performance in properties of this mechanism.

- This approach includes the reduction capacity information which is valuable data to the aggregator in order to plan its aggregated DR capacity for participating in the wholesale electricity market or in bilateral contracts with SO or other agents.

This Chapter is organized as follows. Section 3.1 describes the preliminary setting. In Section 3.2, the problem statement is explained. In Section 3.3, the proposed contract is formulated. In Section 3.4, consumer's optimal choices are developed. In Section 3.5, numerical optimization results are shown. In Section 3.6, a discussion is presented. Final remarks are drawn in Section 3.7. Lastly, mathematical proofs are described in Section 3.8.

3.1 Setting

This Section presents definitions, assumptions and preliminary considerations. This part holds similar setting from previous Chapter, then, this Section is added so that each chapter can be read independently. Incentive-based DR program is managed by an aggregator that requests customers to curtail demand in response to an economic incentive. There are I participant consumers. The set of users is denoted by $\mathcal{I} = \{1, 2, ..., I\}$. Some concepts and conditions are defined as follows.

The decision maker's preferences are specified by giving a utility function $G(q_i; b_i)$, that depicts the level of satisfaction obtained by a user as a function of q_i, which is the energy consumption and b_i is the baseline, which is a quantity only known by each consumer $i \in \mathcal{I}$. The utility function satisfies the following properties as proposed in (Chen et al., 2012):

Property 1. *$G(q_i; b_i)$ is assumed as a concave function with respect to q_i. This implies that the marginal benefit of users is a nonincreasing function, i.e., $\frac{d^2G(q_i;b_i)}{dq_i^2} \leq 0$.*

Property 2. *Utility function is nondecreasing. Therefore, the marginal benefit is nonnegative $\frac{dG(q_i;b_i)}{dq_i} \geq 0$.*

Property 3. *$G(q_i; b_i)$ is zero when the consumption level is zero, $G(0) = 0$.*

The energy price p is given. Then, the following definition are stated.

Definition 9. *The energy total cost is $\pi_i(q_i) = pq_i$.*

Definition 10. *The payoff function without DR is defined as $U_i(q_i; b_i) = G(q_i; b_i) - \pi_i(q_i)$, which indicates the user benefit of consuming q_i for a certain time.*

Definition 11. *The rational behavior of a consumer that maximizes the payoff function $U_i(q_i; b_i)$ is given by, $q_i^* = b_i$. This result is found by solving $q_i^* = \arg\max_{q_i \in \{0, q_{max,i}\}} U_i(q_i; b_i)$. Where $q_{max,i}$ is the maximum allowable consumption value by user i.*

Assumption 1. *Under previous properties, the utility function can be approximated by a second order polynomial. User's utility function is assumed as,*

$$G(q_i; b_i) = \begin{cases} -\frac{\gamma_i}{2}q_i^2 + [\gamma_i b_i + p]q_i & 0 \leq q_i \leq b_i + \frac{p}{\gamma_i} \\ \frac{p^2}{2\gamma_i} + \frac{\gamma_i b_i^2}{2} + pb_i & q_i > b_i + \frac{p}{\gamma_i} \end{cases}$$

where γ_i is the marginal utility of consumer i.

Therefore, given Assumption 1 and Definition 11, the optimal payoff $U_i(q_i; b_i)$ for an agent i that does not participate in DR is equal to

$$\frac{\gamma_i b_i^2}{2} \tag{3.1}$$

Assumption 2. *Each consumer i has a maximum limit of energy consumption $q_{max,i}$. A reasonable hypothesis is that the maximum limit is greater than the saturation limit established in Assumption 1, i.e., $q_{max,i} > b_i + p/\gamma_i$*

3.2 Problem statement

In a wholesale electricity market, according to the available energy sources, aggregated demand information and market prices, SO could require a demand reduction during peak times. Therefore, SO request a DR process through an aggregator. Fig. 3.1 shows the participant agents involved in DR. Aggregator can participate in the market by bidding aggregated reduction capacity to the system. After the market clearing, SO sends a demand reduction requirement to aggregator. Then, by means of contracts with consumers, aggregator activates DR programs.

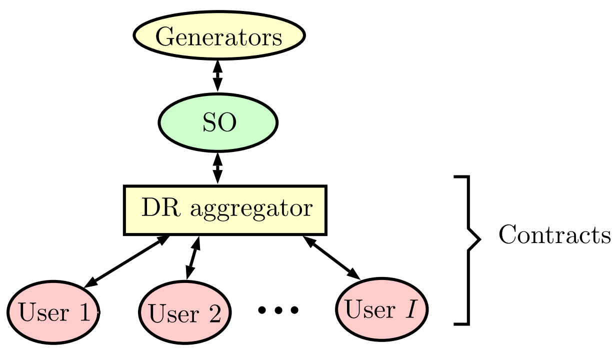

FIGURE 3.1: Participant agents.

In an incentive-based DR approach, aggregator encourages to each participant diminishing the energy pattern according to an economic signal. Mathematically,

Definition 12. *Let p_2 be the rebate price received by the user due to energy reduction in peak periods. The incentive is defined as $\pi_{i,2}(q_i; \hat{b}_i) = p_2(\hat{b}_i - q_i)_+$. Where $(\cdot)_+ = \max(\cdot, 0)$ and the superscript ˆmeans declared information. Therefore, $\hat{b}_i$ is the baseline announced by user.*

Assumption 3. *In order to encourage the energy cutback by demand side. The incentive price is assumed greater or equal than energy price, that is, $p_2 \geq p$.*

In order to obtain an efficient DR service, aggregator requires that each agent reports his true private information (or estimated data on non-altered past consumption) and selects his energy level according to his preferences, i.e.:

$$q_i^o = \underset{q_i \in \{0, q_{max,i}\}}{\arg\max} \; U_i(q_i; b_i) - \pi_i(q_i) + r\pi_{i,2}(q_i; \hat{b}_i) \tag{3.2}$$

with $\hat{b}_i = b_i$ and r_i a binary variable that indicates if user i is called to participate in DR. In other words, truth-telling behavior is desired when incentive-based DR is required during peak times of electrical demand. Under assumption that consumers behave as truthful agents, consumer's optimal choice is given by the following proposition.

Proposition 1. *The optimal consumption q_i^o of a user that participates in an incentive-based DR, given truthful report ($\hat{b}_i = b_i$) is:*

$$q_i^o = \begin{cases} b_i & r_i = 0 \\ (b_i - \frac{p_2}{\gamma_i})_+ & r_i = 1 \end{cases}$$

Case $r_i = 0$ follows from Definition 11. Case $r_i = 1$ is derived from optimality conditions of problem (3.2) at each interval of $G(q_i; b_i)$, and selecting the global maximizer.

Notice that Prop. 1 is the consumer's ideal behavior. However, in practical fashion, a user can alter his reported baseline $\hat{b}_i$, in particular, he could bid $\hat{b}_i = q_{max,i}$ to increase his well-being, without offering a better service to the aggregator and without receiving any penalty. Furthermore, in DR programs like PTR where the baseline is estimated by the aggregator, the optimal strategy of consumers is to inflate the baseline by overconsuming energy during the baseline settling periods, see e.g. (Severin Borenstein, 2014). Moreover, for the case when $p_2 > p$, from Chapter 2, it is shown that overconsumption grows up to $q_{max,i}$. Therefore, a contract between the aggregator and each consumer is proposed to induce asymptotic incentive compatible (truthfulness) and individually rational (voluntary participation) properties.

3.3 Contract based on probability of call

In this section, the proposed contract is described according to a new payment scheme based on probability of call. Fig. 3.2 shows the timeline to hold the contract between the aggregator and an agent. The agreement process is described below.

- First, SO informs to aggregator what net demand reduction is required at a certain peak time or the aggregator could be participating as an energy provider (virtual power plant) within an electricity market. Then aggregator announces energy cost p and incentive price p_2 (Also, this quantity works as penalty for deviation).
- According to given prices, at the time $t-1$, participant consumers (that signed the contract) report their baseline $\hat{b}_i$ and their intended energy consumption $\hat{q}_i$ if they are called.
- Next, aggregator determines the subset of agents that are called to participate in DR subject to SO requirement and probability of call for each participant, i.e., aggregator decides the binary variable r_i.
- Later, at the time t, each user receives his value of r_i and chooses his actual consumption level q_i during the event. This consumption is observed by the aggregator.
- Finally, the incentive payment is made by the aggregator for called users and penalties are charged for deviations.

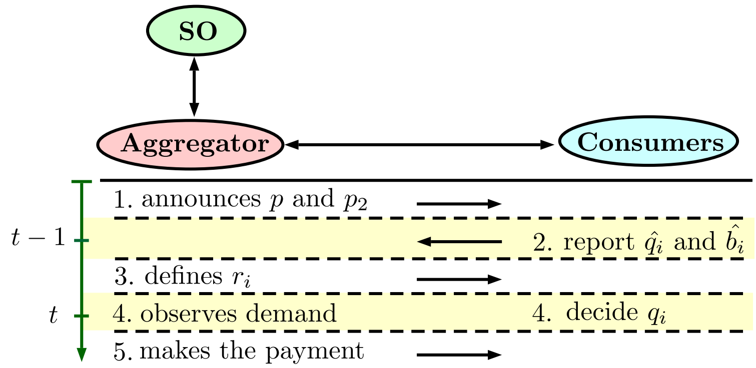

FIGURE 3.2: Timeline of the contract.

The proposed payment scheme of the contract is presented as follows:

Definition 13. *Let $\pi_{i,3}^{r_i}(\hat{b}_i, \hat{q}_i, q_i)$ be the aggregator payment scheme under incentive-based DR:*

$$\pi_{i,3}^{r_i}(\hat{b}_i, \hat{q}_i, q_i) = \begin{cases} p\max(\hat{b}_i, q_i) & r_i = 0 \\ pq_i - p_2(\hat{b}_i - q_i)_+ + p_2\,|q_i - \hat{q}_i| & r_i = 1 \end{cases}$$

Note that if a consumer is not called to participate in DR, then he must pay the maximum between his reported baseline and actual energy consumption. This payment is inspired by the concept of "buy the baseline" (Chao, 2011). Otherwise, if he is required to reduce his energy requirement, then he receives an incentive of $p_2(\hat{b}_i - q_i)$ if he actually reduced consumption and he must pay the energy cost and a penalty with the same price of the incentive if his actual consumption differs from $\hat{q}_i$. Given the payment scheme established in Definition 13, thus the contract is settled as a 2-stage procedure from demand side:
Stage 1) Given the prices p and p_2, each consumer reports $\hat{b}_i$ and $\hat{q}_i$ to aggregator.
Stage 2) Aggregator determines which users are called to participate in DR by means of the variable r_i. Each agent decides his actual energy consumption q_i according to the aggregator decision r_i.
Fig. 3.3 shows the contract sequence where a user faces two decisions at different times. First, the information declaration stage, and next, the DR event. Finally, an important and logical rule of this contract is stated in the following hypothesis.

Assumption 4. *The reported baseline must be greater or equal than reported energy consumption under DR, i.e, $\hat{b}_i \geq \hat{q}_i$.*

In the next Section, consumer's optimal decision under this proposed contract is determined by mathematical proofs. These results are vital for designing the aggregator role.

3.4 Consumer's optimal choices

For the previous DR contract, a consumer faces the problem of deciding what to communicate to aggregator and how much energy q_i to consume. Given p and p_2, a rational

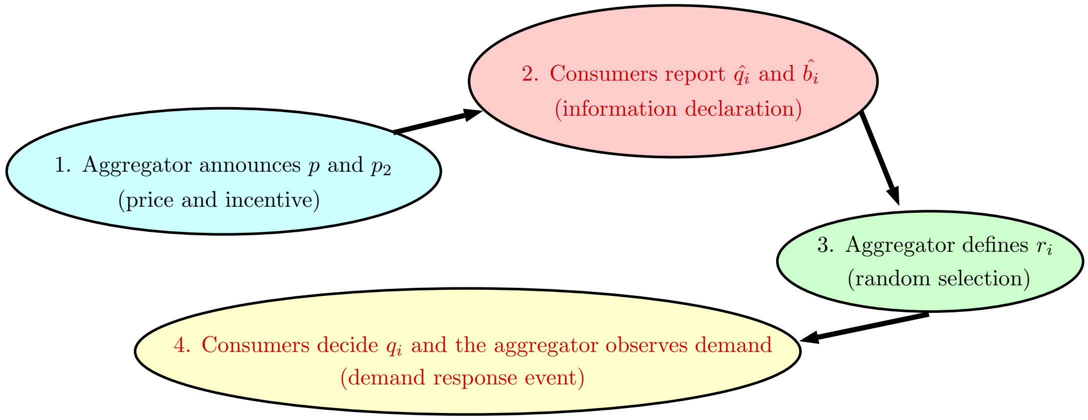

FIGURE 3.3: Scheme of the contract.

user finds $\hat{b}_i$, $\hat{q}_i$ and q_i such that his profits J_i will be maximized:

$$[\hat{b}_i^*, \hat{q}_i^*, q_i^*] = \underset{\hat{b}_i, \hat{q}_i, q_i \in \{0, q_{max,i}\}}{\arg\max} \quad J_i = E(G(q_i; b_i) - \pi_{i,3}^{r_i}(\hat{b}_i, \hat{q}_i, q_i)) \tag{3.3}$$

The superscript * means optimal decisions.
Problem (3.3) is a two-stage stochastic programming formulation. The proposed solution is found by solving backward in time (Rajagopal et al., 2013).
At the time t (see Fig. 3.2), user maximizes profit, by his actual energy consumption q_i for each outcome of r_i, that is, finds the optimal solution $q_{i,r_i=0}^*$ and $q_{i,r_i=1}^*$, given $\hat{b}_i$ and $\hat{q}_i$.

$$[q_{i,r_i=0}^*, q_{i,r_i=1}^*] = \underset{q_{i,r_i} \in \{0, q_{max,i}\}}{\arg\max} \quad G(q_{i,r_i}; b_i) - \pi_{i,3}^{r_i}(\hat{b}_i, \hat{q}_i, q_i) \tag{3.4}$$

At the time $t-1$, consumer determines the best information $\hat{b}_i^*$ and $\hat{q}_i^*$ to report, knowing the optimal choices $q_{i,r_i=0}^*$ and $q_{i,r_i=1}^*$ that he can take at consumption time t and facing the uncertainty in r_i.

$$\begin{aligned}[\hat{b}_i^*, \hat{q}_i^*] = \underset{\hat{b}_i, \hat{q}_i \in \{0, q_{max,i}\}}{\arg\max} \quad E(G(q_{i,r_i}^*; b_i) - \pi_{i,3}^{r_i}(\hat{b}_i, \hat{q}_i, q_{i,r_i}^*)) = \\ \underset{\hat{b}_i, \hat{q}_i \in \{0, q_{max,i}\}}{\arg\max} \quad p_{r_i}[G(q_{i,r_i=1}^*; b_i) - \pi_{i,3}^{r_i=1}(\hat{b}_i, \hat{q}_i, q_{i,r_i=1}^*)] + [1-p_{r_i}][G(q_{i,r_i=0}^*; b_i) - \pi_{i,3}^{r_i=0}(\hat{b}_i, \hat{q}_i, q_{i,r_i=0}^*)]\end{aligned} \tag{3.5}$$

where $p_{r_i} = p_{r_i}(r_i = 1)$ is the probability of call.
The optimal decision problem is described in detail as follows.

3.4.1 Second-stage of consumer's decision

The problem (3.4) is solved for each case of the binary variable r_i. The solutions are presented in Theorems 5 and 6. The proofs are shown in 3.8.1 and 3.8.2, respectively.

Theorem 5. *The optimal consumption q^*_{i,r_i} for the signal $r_i = 0$ of a participant consumer in the proposed contract, given the problem (3.4), is:*

$$q^*_{i,r_i=0} = \begin{cases} b_i & 0 \leq \hat{b}_i \leq b_i \quad \text{strategy } \mathcal{A} \\ \hat{b}_i & b_i < \hat{b}_i \leq b_i + p/\gamma_i \quad \text{strategy } \mathcal{B} \\ b_i + p/\gamma_i & b_i + p/\gamma_i < \hat{b}_i \leq q_{max,i} \quad \text{strategy } \mathcal{C} \end{cases}$$

According to Theorem 5, there are three strategies that depend on the reported baseline. For instance, if a consumer informs a baseline above his preferences and below his saturation limits (i.e. *strategy* $\mathcal{B}$), then, his best choice is to consume what he reported in the previous stage.

Theorem 6. *The optimal consumption q^*_{i,r_i} for the signal $r_i = 1$ of a participant consumer in the proposed contract, given the problem (3.4), is:*

$$q^*_{i,r_i=1} = \begin{cases} b_i & b_i \leq \hat{b}_i \leq q_{max,i}, b_i \leq \hat{q}_i \leq \hat{b}_i \leq q_{max,i} \quad \text{strategy } \mathcal{U} \\ \hat{q}_i & b_i - \frac{p_2}{\gamma_i} \leq \hat{b}_i \leq q_{max,i}, b_i - \frac{2p_2}{\gamma_i} \leq \hat{q}_i \leq \hat{b}_i \leq b_i \quad \text{strategy } \mathcal{V} \\ \hat{q}_i & \alpha \leq \hat{b}_i \leq b_i - \frac{p_2}{\gamma_i}, b_i - \frac{2p_2}{\gamma_i} \leq \hat{q}_i \leq b_i - \frac{p_2}{\gamma_i} \quad \text{strategy } \mathcal{W} \\ (b_i - \frac{p_2}{\gamma_i})_+ & b_i - \frac{2p_2}{\gamma_i} \leq \hat{b}_i \leq \alpha, b_i - \frac{2p_2}{\gamma_i} \leq \hat{q}_i \leq \hat{b}_i \leq b_i - \frac{p_2}{\gamma_i} \quad \text{strategy } \mathcal{X} \\ (b_i - \frac{p_2}{\gamma_i})_+ & 0 \leq \hat{b}_i \leq b_i - \frac{3p_2}{2\gamma_i}, 0 \leq \hat{q}_i \leq \hat{b}_i \leq b_i - \frac{2p_2}{\gamma_i} \quad \text{strategy } \mathcal{Y} \\ (b_i - \frac{2p_2}{\gamma_i})_+ & b_i - \frac{3p_2}{2\gamma_i} \leq \hat{b}_i \leq q_{max,i}, 0 \leq \hat{q}_i \leq b_i - \frac{2p_2}{\gamma_i} \quad \text{strategy } \mathcal{Z} \end{cases}$$

with $\alpha = \frac{\gamma_i b_i^2}{2p_2} - \frac{\gamma_i b_i \hat{q}_i}{p_2} - b_i + \frac{\gamma_i \hat{q}_i^2}{2p_2} + 2\hat{q}_i + \frac{p_2}{2\gamma_i}$.

Similarly, Theorem 6 presents the consumer's optimal decisions conforming to $\hat{b}_i$ and $\hat{q}_i$. In this case, notice that the strategies that enable an energy reduction are *strategies* $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{Z}$. Next, these results are substituted in (3.5) in order to find profit-maximizing behavior.

3.4.2 First-stage of consumer's decision

The solution of problem (3.5) is given in Theorem 7. The proof is described in 3.8.3.

Theorem 7. *Given q^*_{i,r_i} from Theorems 5 and 6, then the optimal reports $\hat{b}_i^*$ and $\hat{q}_i^*$ for (3.5) are:*

$$\hat{b}_i^* = \begin{cases} \frac{p_{r_i} p_2}{\gamma_i (1-p_{r_i})} + b_i & 0 \leq p_{r_i} \leq \frac{p}{p_2+p} \\ q_{max,i} & \frac{p}{p_2+p} \leq p_{r_i} \leq 1 \end{cases}$$

$$\hat{q}_i^* = \left(b_i - \frac{p_2}{\gamma_i}\right)_+$$

Theorem 7 presents the optimal reported decisions for rational consumers in this contract. With respect to the reported baseline $\hat{b}_i^*$, if the probability of call p_{r_i} is less than threshold $p/(p_2 + p)$, then an agent informs his true baseline b_i added to p_2/γ_i which is multiplied by the expression $p_{r_i}/(1 - p_{r_i})$. For instance, if p_{r_i} is close to zero then the best choice is to announce b_i. Therefore, the term $p_{r_i}/(1 - p_{r_i})$ limits the misreporting.

Note that the probability of call is limiting gaming opportunities on the baseline under this agreement. Otherwise, when a user knows that his probability of call exceeds the threshold thus the best strategy is to report his maximum allowed energy $q_{max,i}$. In addition, a consumer decides to inform the energy reduction $\hat{q}_i$, in this affair, the truth information is obtained independently of the probability of call according to Theorem 7.

Collorary 3. *The optimal expected profit J^* is:*

$$J_i^* = \begin{cases} \frac{b_i^2\gamma_i}{2} + \frac{p_{r_i}p_2^2}{2\gamma_i(1-p_{r_i})} & 0 \leq p_{r_i} \leq \frac{p}{p_2+p} \\ \frac{p^2}{2\gamma_i} + b_i p - pq_{max,i} + \frac{b_i^2\gamma_i}{2} - \frac{p^2 p_{r_i}}{2\gamma_i} + \frac{p_2^2 p_{r_i}}{2\gamma_i} & \\ -b_i p_{r_i} - b_i p_2 p_{r_i} + pq_{max,i}p_{r_i} + p_2 q_{max,i} p_{r_i} & \frac{p}{p_2+p} \leq p_{r_i} \leq 1 \end{cases}$$

Corollary 3 is found by substituting solution of Theorem 7 in Eq. (3.3). It is easy to prove that the optimal expected profit J^* is an increasing function w.r.t. p_{r_i}. Relevant cases are presented when the probability of call is lower than the threshold $p/(p_2+p)$. Notice that when p_{r_i} goes to zero, the optimal expected profit under DR is $b_i^2\gamma_i/2$, which is the same value for a user that does not participate in DR as indicated by (3.1).

Collorary 4. *The optimal consumption q_{i,r_i}^*, by replacing the best reports given in Theorem 7, is:*

$$q_{i,r_i}^* = \begin{cases} \frac{p_{r_i}p_2}{\gamma_i(1-p_{r_i})} + b_i & r_i = 0,\ 0 \leq p_{r_i} \leq \frac{p}{p_2+p} \\ b_i + p/\gamma_i & r_i = 0,\ \frac{p}{p_2+p} \leq p_{r_i} \leq 1 \\ (b_i - \frac{p_2}{\gamma_i})_+ & r_i = 1,\ 0 \leq p_{r_i} \leq 1 \end{cases}$$

Lastly, the optimal decision q_{i,r_i}^* is determined by using Theorem 7 in the second-stage defined in Eq. 3.4. On the one hand, if an agent is not called to participate in DR and the probability of call is below the threshold, then he decides to consume the energy he reported in order to earn his best profit. However, if p_{r_i} is above of the threshold $p/(p_2+p)$ when $r_i = 0$, so a rational consumer uses energy to saturation point which is $b_i + p/\gamma_i$. On the other hand, if a user is called to participate $r_i = 1$, then his best choice is to consume what he reported in the previous stage.

3.4.3 Contract properties

Finally, contract properties are direct consequences of previous results. These features are listed below.

Collorary 5. *Individually rational (voluntary participation): a user that participates in this approach obtains a profit at least as good as he does not signing the DR contract.*

An important requirement to encourage affiliation to the contract or mechanism is the property stated in Collorary 5. This results follows by comparing Corollary 3 and expression (3.1). This property means that the participation profit should be equal or greater than not signing the contract. For rational users, other income can be incorporated into their economic activities by participating in proposed settlement.

Collorary 6. *Incentive compatibility on the reported energy consumption under DR: a consumer informs the truthful consumption under DR according to his preferences.*

Collorary 7. *Asymptotic incentive compatibility on the reported baseline: as the probability of call tends to zero, the consumer's optimal strategy is to declare $\hat{b}_i = b_i$.*

Corollaries 6 and 7 establish the truthful properties in this contract. Incentive compatibility refers to the revelation of private information when consumers inform their energy preferences to aggregator. These features are understood by reviewing Theorem 7 and Corollary 4. Therefore, a rational user bids $\hat{q}_i^* = q^*_{i,r_i=1}$ guarantying to maximize his profit. Additionally, if $p_{r_i} \longrightarrow 0$ then the best choice is to announce $\hat{b}_i^* = q^*_{i,r_i=1}$, i.e., asymptotic truthfulness.

3.5 Illustrative example

In this section, simulation results are presented to illustrate the optimal behavior of a user when he is participating in the proposed contract. The retail price is $p = \$0.26/\text{kWh}$ (based on peak summer rate in 10/1/16 by Pacific Gas and Electric Company in San Francisco, California), true baseline is $b_i = 8$ kWh, the incentive/penalty price is $p_2 = \$0.3/\text{kWh}$, the marginal utility is $\gamma_i = \$0.05\ /\text{kWh}^2$ and the maximum allowable consumption is $q_{max,i} = 16$ kWh. Randomness r_i has been created to simulate consumer's optimal choice according to the probability of call. A Monte Carlo simulation is performed with 1000 realizations of r_i for different values of probability in order to check the results.

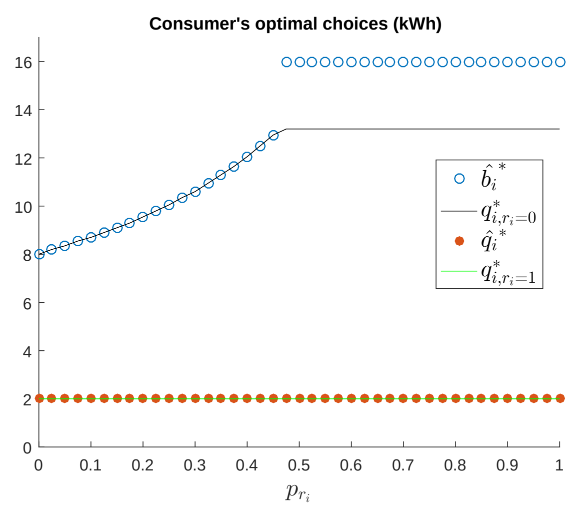

FIGURE 3.4: Optimal consumer's choice.

The problem (3.3) is solved through extensive simulations for different values of probability p_{r_i} to find rational decisions. The consumer's optimal choices are depicted in Fig. 3.4. As regards the reported baseline, the truthfulness is being lost as the probability of call increases which is described by Theorem 7. For this example, the threshold $p/(p+p_2)$ in the probability of call is 0.46. Above the threshold, the worst condition of gaming are found because a user announces his maximum allowed energy consumption and in order to achieve the highest income, as well, he really consumes the maximum energy that generates a benefit when he is not called to participate. Additionally, note that the agent actually declares what he is willing to reduce according to his preferences $\hat{q}_i^* = q^*_{i,r_i=1}$ irrespective of the probability of call because he suffers a penalty for deviations of his self-reported consumption when he is called to DR program.

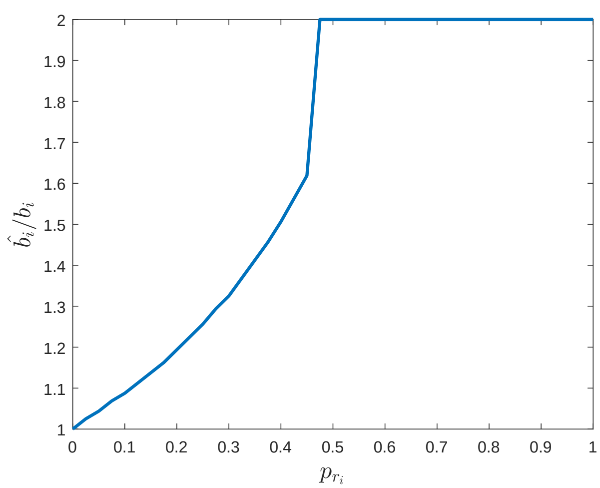

FIGURE 3.5: Percentage of gaming limitation on the reported baseline.

Furthermore, Fig. 3.5 depicts the reported baseline normalized by the true one in order to establish the percentage of limitation on gaming opportunities. For instance, an aggregator calls a group of agents with a probability of call equals to $p_{r_i} = 0.1$ then rational consumers have incentives to overreport the baseline through the increase of 11% due to the expression $p_2 p_{r_i}/(\gamma_i(1-p_{r_i}))$ given by Theorem 7 which is added to his true baseline. Accordingly, an aggregator does not know the values of γ_i for all agents i, however, it can limit their gaming behavior by means of the probability of call.

The intuition behind these findings is that, on the one hand, whether the participant user is not called then he must pay the maximum between his reported baseline and actual consumption, thus, misreporting will cause a loss of his profit. On the other hand, if the consumer is called, hence he should reduce his energy consumption according to what he committed at previous Stage so that the penalty does not apply. Moreover, consumer's optimal decisions are coherent with Prop. 1 given a probability of call close to zero. Therefore, asymptotic incentive compatibility is induced by means of this contract.

Fig. 3.6 shows the optimal expected profit of a consumer according to the probability of call. This curve is increasing with p_{r_i} and two parts are distinguished as is stated in Corollary 3. The first one, a smooth growth is exhibited for $p_{r_i} \in [0, 0.46]$ then a significant rise in benefits is presented for $p_{r_i} > 0.46$ as the second part. The point of change between the two parts is presented in the threshold probability of call, which is coherent with Fig. 3.4. Furthermore, the payoff when an agent does not participate in DR is given by expression (3.1) that for this case is \$1.6. In Fig. 3.6, the lowest optimal expected profit occurs in $p_{r_i} = 0$ with a payoff of \$1.6, hence, the user's benefit when he participates in this contract is at least as good as when he does not join in the

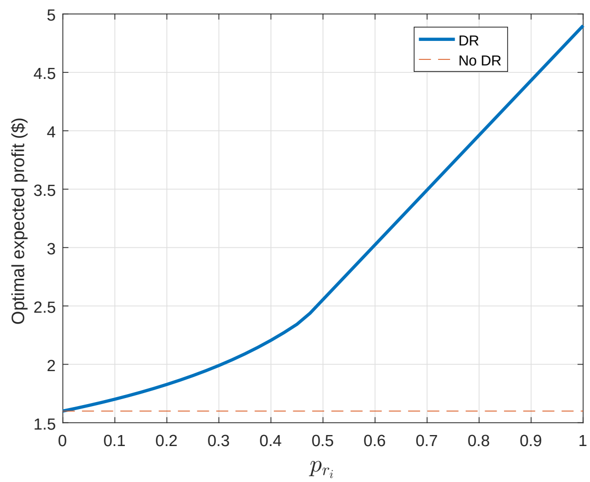

FIGURE 3.6: Optimal expected profit of a consumer.

incentive-based DR program. Lastly, voluntary participation, also known as individual rationality, is motivated in the proposed contract.

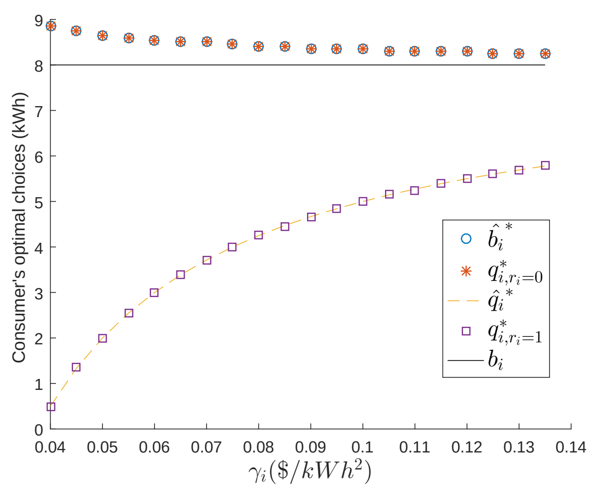

FIGURE 3.7: consumer's optimal choice and optimal profit by varying private preference γ_i given the probability of call $p_{r_i} = 0.1$.

Finally, variations in the preference γ_i are analyzed in this contract. In Fig. 3.7, the marginal utility γ_i is changed given fixed probability of call and prices, in order to study the effects on the fluctuation of this consumer's private preference. For $p_{r_i} = 0.1$, the optimal reported baseline $\hat{b}_i^*$ is equal to the actual energy consumption $q^*_{i,r_i=0}$ if he is not called for DR and also, as γ_i increases, the reported baseline tends to the true baseline b_i. In addition, reported and actual energy consumption are the same $\hat{q}_i^* = q^*_{i,1}$, which is consistent to Theorem 7 and Corollary 4. In summary, as electrical energy preference γ_i rises for a probability of call beneath of threshold value, overbidding in the baseline and reduction capacity diminish; also, the reported information and the actual consumption values hold the same behavior.

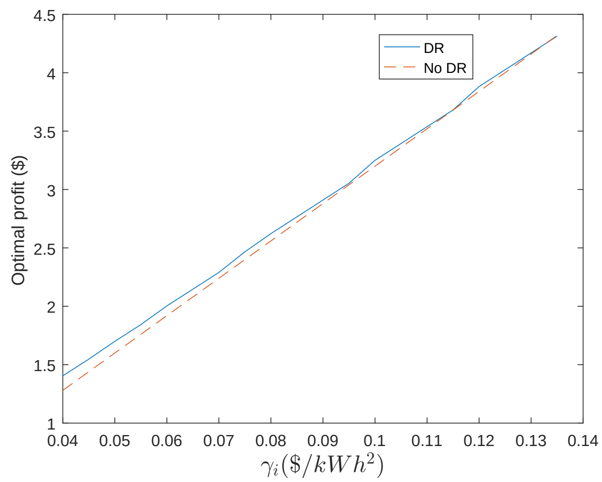

FIGURE 3.8: Optimal profit by varying private preference γ_i given the probability of call $p_{r_i} = 0.1$.

Lastly, Fig. 3.8 presents the optimal profit by changing γ_i. The curve trend is an increasing function with respect to γ_i due to preliminary assumptions of the consumer behavior. Note that the property of voluntary participation is guaranteed irrespective of the consumer's preferences since a rational user (profit-maximizing) obtains greater economic profit by joining in DR rather than not participating.

3.6 Discussion

The probability of call means the chance of a consumer to be selected by the aggregator to serve as DR resource at a given period. This work is developed from the demand side, resulting in that every consumer submits the baseline according to the probability of call which is managed by the aggregator. If a user is called, then he performs DR but if he is not called then he faces a penalty if there is an over-reporting. Therefore, under this contract, a user faces the uncertainty of the call, thus, the aggregator should call randomly agents according to the probability of call to limit baseline alteration from the demand side.

Traditional incentive-based DR programs rely on counter-factual models to estimate baseline in order to make the payments to participant users. This approach is vulnerable for gaming since consumers increase their energy consumption during baseline setting period harming the system reliability. The proposed contract induces an asymptotic incentive compatible property on the self-reported baseline by participant agents since the marginal utility is not available to settle the contract. In spite of this agreement feature, a regular consumer is not taking decisions with stochastic optimization tools, therefore, whether the aggregator keeps the probability of call close enough to zero then the truth-telling behavior is obtained under this novel approach.

This contract can be compared with the PTR program. In PTR, a residential consumer receives a rebate according to a forecasted baseline during peak period times. Participant users are notified in advance that DR process is required. Therefore, a consumer is certain that he is going to participate in the program. Then he can alter the baseline setting in order to improve his economic benefits. There are some celebrated cases, e.g., in (Severin Borenstein, 2014), and the previous Chapter quantifies the gaming behavior of a consumer under PTR. In the proposed contract, a user faces the uncertainty of being called or not in order to control the baseline alteration. The disadvantage is that the truthfulness depends on the probability of call.

In practical fashion, an industrial, commercial or residential consumer is not thinking in terms of marginal utility when he is dealing with DR. In most solutions found in the literature, the agreement or mechanism requires that agents reveal all their types including the private value γ_i in order to settle the problem. It is most intuitively that a user with smart metering devices can know or estimate how much energy he consumes during a certain hour of the day, thus he could announce information to the aggregator in terms of kWh. For instance, if an industrial consumer is willing to turn off some machines during DR event then he can determine his possible reduction of energy through knowledge of the equipment power. Therefore, a user with the suitable technology that analyzes the behavior of each appliance can take his decisions and participate in incentive-based DR programs by knowing his preferences of energy consumption. One of the advantages of the proposed contract is actually that the required information of participants is in energy units. Although this new approach includes the analysis of marginal utility, the contract can be implemented without this knowledge.

For the implementation, energy monitor is required for every consumer to follow the consumption in real time. This information should be available for consumers and aggregator. An application software can be implemented as a tool in order to exchange information between agents. Participant users should know their energy consumption behavior, e.g., if a consumer is aware of the energy pattern of certain equipment in kWh and he usually uses it during the time of peak event, then, he can bid the baseline (regular consumption without DR) and he can decide to turn off that appliance during DR process. Therefore, that user can inform the amount of kWh that he is willing to reduce according to his load. That information could be submitted to the software and later, aggregator informs if that consumer was selected to perform DR (Probability of call).

According to Corollary 6, incentive compatibility on the reported energy consumption under DR is guaranteed due to the form of the penalty function which is given by a expression with absolute value, i.e., $p_2 |q_i - \hat{q}_i|$. At first sight, it might seem that this penalty is very strict. Although, it is possible to relax this condition through tolerance or hysteresis band in order to incorporate some flexibility in the contract and that agents do not feel so rigid when consuming energy at Stage 2 if they are called to participate in DR. The final goal is to design a contract that could be implementable and elicit truthful properties for improving part of the power management in a smart grid.

3.7 Remarks

In this Chapter was proposed a demand response contract that induces voluntary participation and asymptotic truthful properties based on the probability of call for users.

In addition, this approach is susceptible to be implemented since this contract uses information in terms of energy which can be obtained by power monitors. The problem was addressed using a two-stage stochastic programming algorithm. The formulation allowed linking the consumer's decisions between the different stages in order to determine the consumer's optimal behavior. It was found that if an aggregator keeps the probability of call close to zero then consumers reveal their truth information about energy preferences. Hence, the main goal of the aggregator is to call randomly a subset of users that meets the probability criterion and satisfies the energy reduction requirement demand by system operator.

A contract for incentive-based demand response based on probability of call enables to limit the gaming opportunities by making that a consumer buys his self-reported baseline or pays his actual consumption if this one is greater than his bid when he is not called to participate in demand response. The user's uncertainty under this agreement motivates him to inform the truth of his preferences. In addition, the model provides to the aggregator the limit allowed probability to maintain a bounded deviation in user behavior. Therefore, the limitation of gaming occurs beneath of threshold and its percentage is controlled by the probability of call.

3.8 Mathematical proofs

3.8.1 Proof of the Theorem 5

Proof. The optimization problem is analyzed by cases due to the non-linearities in the functions. Then several solutions are found according to the reported information given by a user at Stage 1. Finally, results are compared and then the global solution is determined. Considering the signal $r_i = 0$, next, the optimization problem is posed as follows:

$$q^*_{i,r_i=0} = \underset{q_{i,r_i=0} \in \{0, q_{max,i}\}}{\arg\max} \quad H_{r_i=0} = G(q_{i,r_i}; b_i) - p \max(\hat{b}_i, q_i)$$

Four cases are identified as results of payment function which is the maximum between reported baseline and actual energy consumption, and the two parts of utility functions $G(.)$ (see Def. 1). Cases are denoted by lowercase letters. A optimization problem is formulated for each case according to its conditions.

The case a) is assumed when reported baseline is greater than the actual consumption, and the utility function is non-saturated, i.e.,

a) $r_i = 0, \hat{b}_i \geq q_i, 0 \leq q_i \leq b_i + \frac{p}{\gamma_i}, 0 \leq \hat{b}_i \leq q_{max,i}$

$$\begin{aligned} q^*_{i,r_i=0} = \quad & \underset{q_{i,r_i=0}}{\arg\max} \quad H_{r_i=0} = -\frac{\gamma_i}{2} q_i^2 + [\gamma_i b_i + p] q_i - p\hat{b}_i \\ & \text{s.t.} \qquad -\hat{b}_i + q_i \leq 0 \\ & \qquad\qquad q_i - b_i - p/\gamma_i \leq 0 \\ & \qquad\qquad -q_i \leq 0 \end{aligned}$$

The first-order optimality condition yields to

$$q^*_{i,r_i=0} = \begin{cases} \hat{b}_i & 0 \le \hat{b}_i \le b_i + p/\gamma_i \\ b_i + p/\gamma_i & b_i + p/\gamma_i \le \hat{b}_i \le q_{max,i} \end{cases}$$

Next, the optimal payoff for this case is given by

$$H^*_{r_i=0} = \begin{cases} -\frac{\hat{b}_i^{\,2}\gamma_i}{2} + b_i\hat{b}_i\gamma_i & 0 \le \hat{b}_i \le b_i + p/\gamma_i \\ \frac{p^2}{2\gamma_i} + \frac{\gamma_i b_i^2}{2} + p(b_i - \hat{b}_i) & b_i + p/\gamma_i \le \hat{b}_i \le q_{max,i} \end{cases} \tag{3.6}$$

Then the same condition but the utility function is saturated.
b) $r_i = 0$, $\hat{b}_i \ge q_i$, $q_i > b_i + \frac{p}{\gamma_i}$, $q_i \le q_{max,i}$, $0 \le \hat{b}_i \le q_{max,i}$

$$\begin{aligned} q^*_{i,r_i=0} = \quad & \underset{q_{i,r_i=0}}{\arg\max} \quad H_{r_i=0} = \frac{p^2}{2\gamma_i} + \frac{\gamma_i b_i^2}{2} + pb_i - p\hat{b}_i \\ & \text{s.t.} \qquad -\hat{b}_i + q_i \le 0 \\ & \qquad\qquad -q_i + b_i + p/\gamma_i \le 0 \\ & \qquad\qquad q_i - q_{max,i} \le 0 \end{aligned}$$

The first-order optimality condition yields to

$$q^*_{i,r_i=0} \in [b_i + p/\gamma_i, q_{max,i}], \quad b_i + p/\gamma_i \le \hat{b}_i \le q_{max,i}$$

Later, the optimal payoff for this case results to

$$H^*_{r_i=0} = \frac{p^2}{2\gamma_i} + \frac{\gamma_i b_i^2}{2} + p(b_i - \hat{b}_i), \quad b_i + p/\gamma_i \le \hat{b}_i \le q_{max,i} \tag{3.7}$$

The case c) is assumed when reported baseline is lower than the actual consumption, and the utility function is non-saturated part, i.e.,
c) $r_i = 0$, $\hat{b}_i \le q_i$, $0 \le q_i \le b_i + \frac{p}{\gamma_i}$, $0 \le \hat{b}_i \le q_{max,i}$

$$\begin{aligned} q^*_{i,r_i=0} = \quad & \underset{q_{i,r_i=0}}{\arg\max} \quad H_{r_i=0} = -\frac{\gamma_i}{2}q_i^2 + [\gamma_i b_i + p]q_i - pq_i \\ & \text{s.t.} \qquad \hat{b}_i - q_i \le 0 \\ & \qquad\qquad q_i - b_i - p/\gamma_i \le 0 \\ & \qquad\qquad -q_i \le 0 \end{aligned}$$

The first-order optimality condition yields to

$$q^*_{i,r_i=0} = \begin{cases} b_i & 0 \le \hat{b}_i \le b_i \\ \hat{b}_i & b_i \le \hat{b}_i \le b_i + p/\gamma_i \end{cases}$$

Next, the optimal payoff for this case is given by

$$H^*_{r_i=0} = \begin{cases} \frac{\gamma_i b_i^2}{2} & 0 \le \hat{b}_i \le b_i \\ -\frac{\hat{b}_i^2 \gamma_i}{2} + b_i \hat{b}_i \gamma_i & b_i \le \hat{b}_i \le b_i + p/\gamma_i \end{cases} \tag{3.8}$$

Finally, the last case is when reported baseline is lower than the actual consumption and the utility function is saturated.

d) $r_i = 0$, $\hat{b}_i \le q_i$, $q_i > b_i + \frac{p}{\gamma_i}$, $q_i \le q_{max,i}$, $0 \le \hat{b}_i \le q_{max,i}$

$$\begin{aligned} q^*_{i,r_i=0} = \quad & \underset{q_{i,r_i=0}}{\arg\max} \quad H_{r_i=0} = \frac{p^2}{2\gamma_i} + \frac{\gamma_i b_i^2}{2} + pb_i - pq_i \\ & \text{s.t.} \qquad \hat{b}_i - q_i \le 0 \\ & \qquad\quad\ - q_i + b_i + p/\gamma_i \le 0 \\ & \qquad\quad\ q_i - q_{max,i} \le 0 \end{aligned}$$

The first-order optimality condition yields to

$$q^*_{i,r_i=0} = b_i + p/\gamma_i, \quad 0 \le \hat{b}_i \le b_i + p/\gamma_i$$

And the optimal payoff results to

$$H^*_{r_i=0} = -\frac{p^2}{2\gamma_i} + \frac{\gamma_i b_i^2}{2}, \quad 0 \le \hat{b}_i \le b_i + p/\gamma_i \tag{3.9}$$

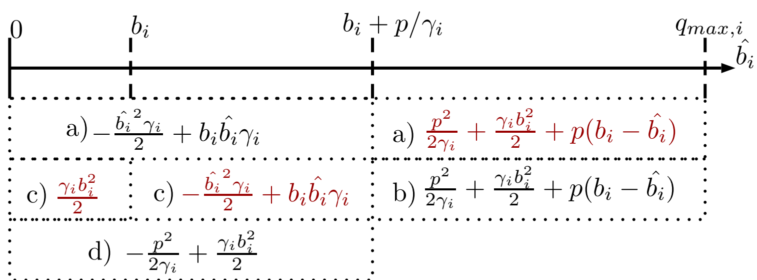

FIGURE 3.9: Comparison of optimal profits at Stage 2 when $r_i = 0$.

Fig. 3.9 shows the comparison of optimal payoff (3.6), (3.7), (3.8) and (3.9). The final solution is found by selecting the maximum profit according to the reported baseline. Red expressions in Fig. 3.9 corresponds to maximum values.

Finally, the optimal decision at the Stage 1 is drawn in Fig. 3.10. The result is given by Theorem 5.

□

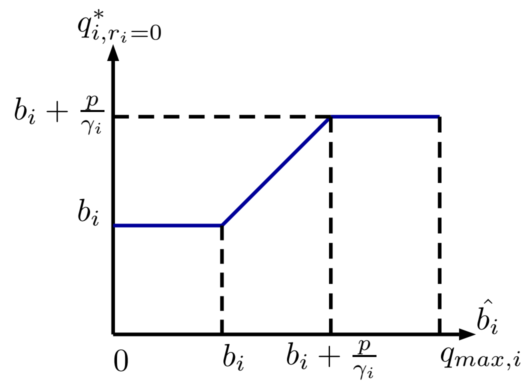

FIGURE 3.10: Optimal solution at Stage 1 for $r_i = 0$.

3.8.2 Proof of the Theorem 6

Proof. The procedure to solve the Stage 2 when $r_i = 1$ is similar to Theorem 5. Although, the results are given by the reported baseline and the reduced energy consumption under DR. The optimization problem is written as follows:

$$q^*_{i,r_i=1} = \underset{q_{i,r_i=1} \in \{0, q_{max,i}\}}{\arg\max} \quad H_{r_i=1} = G(q_{i,r_i}; b_i) - [pq_i - p_2(\hat{b}_i - q_i)_+ + p_3 |q_i - \hat{q}_i|]$$

Since the problem is nonlinear then eight cases are identified but six of them are feasible due to Assumption 4. Two parts of utility function $G(.)$, two combinations for incentive expression, and two regions in the penalty of deviation are taken into account in this analysis. All cases and their solutions are listed below.

e) $r_i = 1$, $q_i \neq \hat{q}_i$, $0 \leq q_i \leq b_i + p/\gamma_i$, $q_i \geq \hat{b}_i$, $q_i - \hat{q}_i > 0$, $0 \leq \hat{b}_i \leq q_{max,i}$, $0 \leq \hat{q}_i \leq q_{max,i}$

$$q^*_{i,r_i=1} = \begin{cases} (b_i - \frac{p_2}{\gamma_i})_+ & 0 \leq \hat{b}_i \leq b_i - \frac{p_2}{\gamma_i}, 0 \leq \hat{q}_i \leq b_i - \frac{p_2}{\gamma_i} \qquad e)\,1 \\ \hat{b}_i & b_i - \frac{p_2}{\gamma_i} \leq \hat{b}_i \leq b_i + \frac{p}{\gamma_i}, 0 \leq \hat{q}_i \leq \hat{b}_i \qquad e)\,2 \end{cases}$$

$$H^*_{r_i=1} = \begin{cases} \frac{b_i^2 \gamma_i}{2} - b_i p_2 + \frac{p_2^2}{2\gamma_i} + \hat{q}_i p_2 & 0 \leq \hat{b}_i \leq b_i - \frac{p_2}{\gamma_i}, 0 \leq \hat{q}_i \leq b_i - \frac{p_2}{\gamma_i} \qquad e)\,1 \\ p_2 \hat{q}_i - \hat{b}_i p_2 - \frac{\hat{b}_i^2 \gamma_i}{2} + b_i \hat{b}_i \gamma_i & b_i - \frac{p_2}{\gamma_i} \leq \hat{b}_i \leq b_i + \frac{p}{\gamma_i}, 0 \leq \hat{q}_i \leq \hat{b}_i \qquad e)\,2 \end{cases} \tag{3.10}$$

f) $r_i = 1$, $q_i \neq \hat{q}_i$, $0 \leq q_i \leq b_i + p/\gamma_i$, $q_i \leq \hat{b}_i$, $q_i - \hat{q}_i > 0$, $0 \leq \hat{b}_i \leq q_{max,i}$, $0 \leq \hat{q}_i \leq q_{max,i}$

$$q^*_{i,r_i=1} = \begin{cases} (b_i - \frac{2p_2}{\gamma_i})_+ & b_i - \frac{2p_2}{\gamma_i} \leq \hat{b}_i \leq q_{max,i}, 0 \leq \hat{q}_i \leq b_i - \frac{2p_2}{\gamma_i} \qquad f)\,1 \\ \hat{b}_i & 0 \leq \hat{b}_i \leq b_i - \frac{2p_2}{\gamma_i}, 0 \leq \hat{q}_i \leq \hat{b}_i \leq b_i - \frac{2p_2}{\gamma_i} \qquad f)\,2 \\ \hat{q}_i & 0 \leq \hat{q}_i \leq \hat{b}_i \leq q_{max,i}, b_i - \frac{2p_2}{\gamma_i} \leq \hat{q}_i \leq b_i + \frac{p}{\gamma_i} \qquad f)\,3 \end{cases}$$

$$H^*_{r_i=1} = \begin{cases} \frac{2p_2^2}{\gamma_i} - 2b_ip_2 + \hat{b}_ip_2 + p_2\hat{q}_i + \frac{b_i^2\gamma_i}{2} & b_i - \frac{2p_2}{\gamma_i} \leq \hat{b}_i \leq q_{max,i}, 0 \leq \hat{q}_i \leq b_i - \frac{2p_2}{\gamma_i} \qquad f)\,1 \\ p_2\hat{q}_i - \hat{b}_ip_2 - \frac{\hat{b}_i^2\gamma_i}{2} + b_i\hat{b}_i\gamma_i & 0 \leq \hat{b}_i \leq b_i - \frac{2p_2}{\gamma_i}, 0 \leq \hat{q}_i \leq \hat{b}_i \leq b_i - \frac{2p_2}{\gamma_i} \qquad f)\,2 \\ \hat{b}_ip_2 - p_2\hat{q}_i - \frac{\hat{q}_i^2\gamma_i}{2} + b_i\hat{q}_i\gamma_i & 0 \leq \hat{q}_i \leq \hat{b}_i \leq q_{max,i}, b_i - \frac{2p_2}{\gamma_i} \leq \hat{q}_i \leq b_i + \frac{p}{\gamma_i} \qquad f)\,3 \end{cases} \tag{3.11}$$

g) $r_i = 1$, $q_i \neq \hat{q}_i$, $q_i \geq b_i + p/\gamma_i$, $q_i \geq \hat{b}_i$, $q_i \leq q_{max,i}$, $q_i - \hat{q}_i > 0$, $0 \leq \hat{b}_i \leq q_{max,i}$, $0 \leq \hat{q}_i \leq q_{max,i}$

$$q^*_{i,r_i=1} = b_i + p/\gamma_i \quad 0 \leq \hat{b}_i \leq b_i + p/\gamma_i, 0 \leq \hat{q}_i \leq b_i + p/\gamma_i$$

$$H^*_{r_i=1} = \frac{\gamma_i b_i^2}{2} - p_2b_i - \frac{p^2}{2\gamma_i} - \frac{p_2p}{\gamma_i} + p_2\hat{q}_i \tag{3.12}$$

h) $r_i = 1$, $q_i \neq \hat{q}_i$, $q_i \geq b_i + p/\gamma_i$, $q_i \leq \hat{b}_i$, $q_i \leq q_{max,i}$, $q_i - \hat{q}_i > 0$, $0 \leq \hat{b}_i \leq q_{max,i}$, $0 \leq \hat{q}_i \leq q_{max,i}$

$$q^*_{i,r_i=1} = b_i + p/\gamma_i \quad b_i + p/\gamma_i \leq \hat{b}_i \leq q_{max,i}, 0 \leq \hat{q}_i \leq b_i + p/\gamma_i$$

$$H^*_{r_i=1} = \frac{\gamma_i b_i^2}{2} - 2p_2b_i - \frac{p^2}{2\gamma_i} - \frac{2p_2p}{\gamma_i} + \hat{b}_ip_2 + p_2\hat{q}_i \tag{3.13}$$

i) $r_i = 1$, $q_i \neq \hat{q}_i$, $0 \leq q_i \leq b_i + p/\gamma_i$, $q_i \geq \hat{b}_i$, $q_i - \hat{q}_i < 0$, $0 \leq \hat{b}_i \leq q_{max,i}$, $0 \leq \hat{q}_i \leq q_{max,i}$. Case i) is infeasible by Assumption 4.

j) $r_i = 1$, $q_i \neq \hat{q}_i$, $0 \leq q_i \leq b_i + p/\gamma_i$, $q_i \leq \hat{b}_i$, $q_i - \hat{q}_i < 0$, $0 \leq \hat{b}_i \leq q_{max,i}$, $0 \leq \hat{q}_i \leq q_{max,i}$

$$q^*_{i,r_i=1} = \begin{cases} b_i & b_i \leq \hat{b}_i \leq q_{max,i}, b_i \leq \hat{q}_i \leq q_{max,i} \qquad j)\,1 \\ \hat{q}_i & \hat{q}_i \leq \hat{b}_i \leq q_{max,i}, 0 \leq \hat{q}_i \leq b_i \qquad j)\,2 \end{cases}$$

$$H^*_{r_i=1} = \begin{cases} \hat{b}_ip_2 - p_2\hat{q}_i + \frac{b_i^2\gamma_i}{2} & b_i \leq \hat{b}_i \leq q_{max,i}, b_i \leq \hat{q}_i \leq q_{max,i} \qquad j)\,1 \\ \hat{b}_ip_2 - p_2\hat{q}_i - \frac{\hat{q}_i^2\gamma_i}{2} + b_i\hat{q}_i\gamma_i & \hat{q}_i \leq \hat{b}_i \leq q_{max,i}, 0 \leq \hat{q}_i \leq b_i \qquad j)\,2 \end{cases} \tag{3.14}$$

k) $r_i = 1$, $q_i \neq \hat{q}_i$, $q_i \geq b_i + p/\gamma_i$, $q_i \geq \hat{b}_i$, $q_i \leq q_{max,i}$, $q_i - \hat{q}_i < 0$, $0 \leq \hat{b}_i \leq q_{max,i}$, $0 \leq \hat{q}_i \leq q_{max,i}$. As well, case k) is infeasible.

Finally, case l) is solved.

l) $r_i = 1$, $q_i \neq \hat{q}_i$, $q_i \geq b_i + p/\gamma_i$, $q_i \leq \hat{b}_i$, $q_i \leq q_{max,i}$, $q_i - \hat{q}_i < 0$, $0 \leq \hat{b}_i \leq q_{max,i}$, $0 \leq \hat{q}_i \leq q_{max,i}$.

$$q^*_{i,r_i=1} = b_i + p/\gamma_i \quad b_i + p/\gamma_i \leq \hat{b}_i \leq q_{max,i}, b_i + p/\gamma_i \leq \hat{q}_i \leq q_{max,i}$$

$$H^*_{r_i=1} = \frac{\gamma_i b_i^2}{2} - \frac{p^2}{2\gamma_i} + \hat{b}_ip_2 - p_2\hat{q}_i \tag{3.15}$$

Next, there are two local maxima when solution e) 1 and f) 1 are found in the same feasible region. In order to find the global solution, the payoff in cases e) 1 and f) 1 are compared. The critical value of $\hat{b}_i$ that provides the same payoff in both cases is:

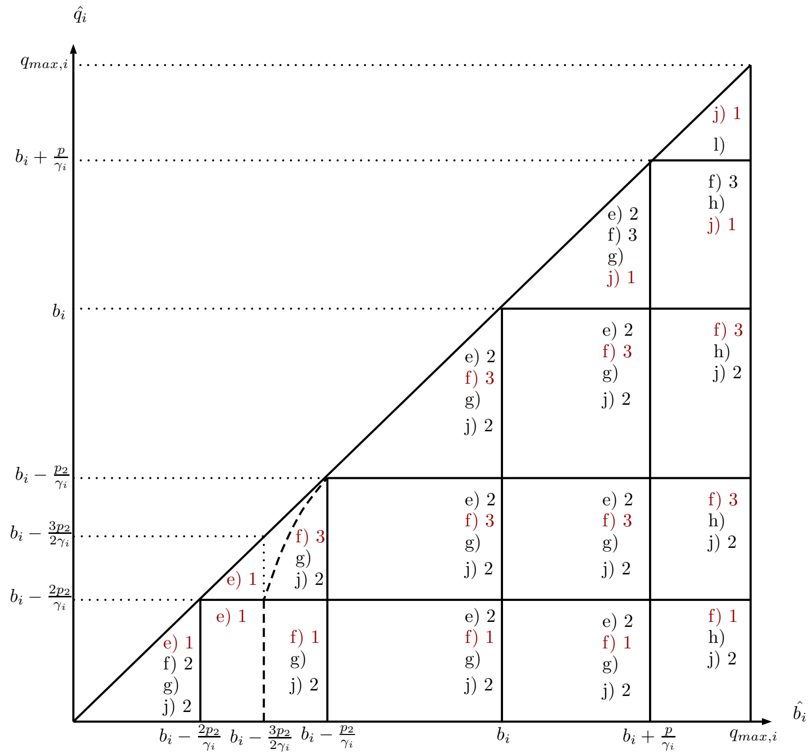

FIGURE 3.11: Comparison of optimal profits at Stage 2 when $r_i = 1$.

$\frac{b_i^2\gamma_i}{2} - b_i p_2 + \frac{p_2^2}{2\gamma_i} + \hat{q}_i p_2 = \frac{2p_2^2}{\gamma_i} - 2b_i p_2 + \hat{b}_i p_2 + p_2\hat{q}_i + \frac{b_i^2\gamma_i}{2}$,
Then solving for $\hat{b}_i$,
$\hat{b}_i = b_i - \frac{3p_2}{2\gamma_i}$
Similarly, the same situation occurs for e) 1 and f) 3. The critical value is renamed as α because depends on reported baseline $\hat{b}_i$ and the announce of reduced energy consumption $\hat{q}_i$. This value is found in Theorem 6.

Fig. 3.11 presents the comparison of optimal payoff (3.10), (3.11), (3.12), (3.13), (3.14) and (3.15). The final solution is found by choosing the maximum profit according to $\hat{b}_i$ and $\hat{q}_i$. Red expression in Fig. 3.11 corresponds to maximum values. The result is given by Theorem 6. □

3.8.3 Proof of the Theorem 7

Proof. The results from Theorems 5 and 6 are substituted in Eq. (3.5) in order to solve the optimization problem. Fig. 3.12 presents the intersection of strategies at Stage 2. Every crossing between results when $r_i = 0$ and $r_i = 1$ in (3.5) is solved. Next, the outcomes are compared according to the reported baseline and reduced energy consumption in order to determine the global maximum. The final result is given by Theorem 7.

□

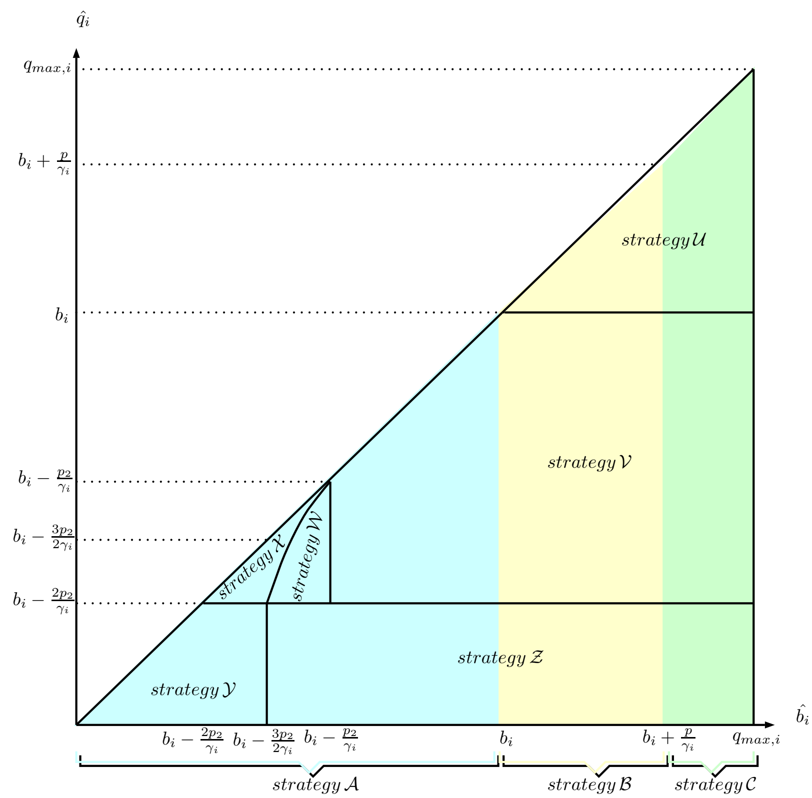

FIGURE 3.12: Intersection of feasible regions.

Chapter 4

A dynamic pricing model for electric vehicle fleet operators

Electric vehicles have become an essential part of the current grid. Nevertheless, at the distribution level, the additional demand generated by the growing number of EV may have adverse impacts on the grid due to undesirable conditions during the charging process (Gruosso, 2017a; Gruosso, 2017b). This issue raises new challenges for the power system operation in terms of smart charging management so it becomes necessary the appearance of a new market agent known as demand aggregators or EV fleet operators. In Chapter 3 was presented a contract between an aggregator and a consumer to face gaming behavior under incentive-based DR programs. In this Chapter is proposed an EV aggregator contract based on a model of the interaction between a fleet operator and electric vehicles under a dynamic pricing program or a price-based contract. A literature review about EV charging solutions was described in Subsection 1.1.3 showing that more research in price model is required. The proposed model allows the determination of the price signal delivering maximum aggregator profit, and the optimal load pattern from EVs under the proposed solution. An assumption, similar to (Zugno et al., 2013), is considered in which the two parties are agreed on certain characteristics of a variable electricity price, i.e. minimum, maximum and average value during the day. An MPEC optimization problem is formulated to model the interaction between the involved agents. At the upper-level, aggregator maximizes its profit whereas the lower-level represents the behavior of rational EV drivers as a fleet. In addition, uncertainties are included by considering a scenario-probability framework in the model. MPEC formulation is transformed into a MILP Algorithm that can be solved in a commercial optimization software. This model can be used as price planner for indirect methods of charging management for EVs. The key points are summarized as follows:

- A game theoretic approach is proposed to model the interaction between an aggregator and flexible EV owners to design retail tariffs by facing uncertainties in demand and spot prices.
- The consumer behavior is depicted through an optimization problem rather than using demand elasticities or utility functions for EV applications.
- By using an aggregated virtual battery formulation as a state-space model, the dynamics of consumer behavior is captured in the lower-level of the proposed bilevel game.

This Chapter is structured as follows. Section 4.1 describes the problem. Section 4.2 introduces the mathematical formulation. In Section 4.3, Numerical results are shown. Remarks are drawn in Section 4.4.

4.1 Problem description

In this setting, the economic optimization problem of an EV fleet operator is considered, which is an intermediate agent between wholesale electricity markets and EV owners. An aggregator is responsible of providing charging services to EV fleet and to manage its customers for other purposes, e.g., ancillary services and balancing operations. Unidirectional charging approach is considered, in which price signals are used as the control strategy as shown in Fig. 4.1. Energy is purchased at the electricity market (e.g. at the day-ahead stage) and then it is sold to EV owners, who face an optimization problem aimed at minimizing the cost of their energy consumption.

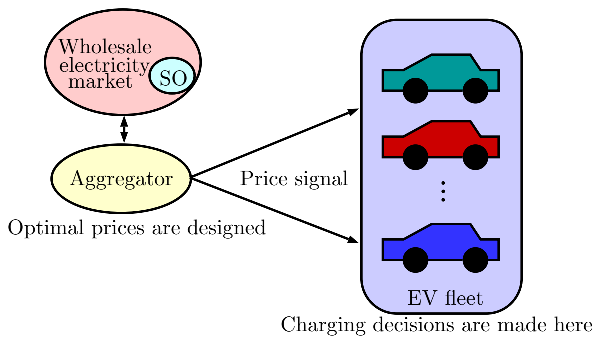

FIGURE 4.1: Information flow between aggregator and EV fleet based on price strategy.

From the perspective of the decision-making process, the aggregator has to determine the price signals so that the users then decide their consumption pattern according to price values. This sequential interaction is captured by a dynamic game, known as a Stackelberg structure (Vega Redondo, 2003), which is formulated as a bilevel optimization problem. This model is depicted through an MPEC algorithm. An advantage of this formulation is that EV fleet behavior is incorporated in the model as an optimization problem presenting alternatives to solutions that require to choose demand elasticities (Yu, Yang, and Rahardja, 2012) or consumer benefit functions (Yoon et al., 2016). Furthermore, given that the aggregator goal is to act as a price planner, note that at the time when the problem is solved, EV fleet operator does not know either the values of the spot prices nor the energy requirements of the EV owners. Therefore, two random variables are identified in this problem.

The proposed agreement is depicted in Fig. 4.2, which is a decision sequence diagram of an EV fleet operator under this setting. Previously, the aggregator and EV fleet are agreed on certain parameters of a variable electricity price: minimum $\underline{\gamma}$, maximum $\overline{\gamma}$ and average value $\hat{\gamma}$ during the day. The contract process is described below.

- First, aggregator designs retail prices according to EV behavior by maximizing its expected profit considering uncertainties on demand and spot prices.

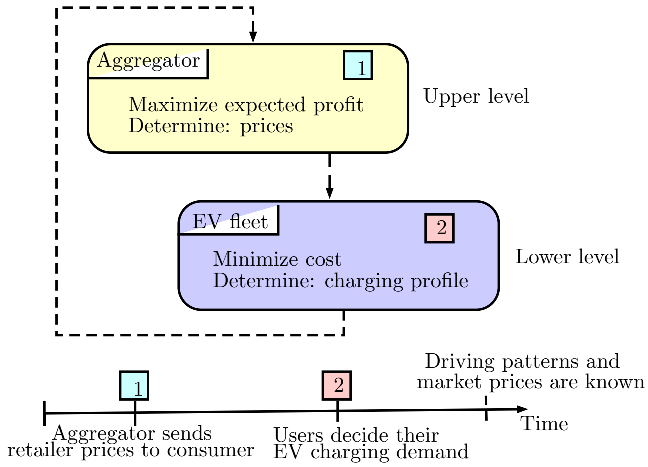

FIGURE 4.2: Decision-making process and bilevel model.

- Second, the EV fleet decides its consumption curve by defining the charging profile given the prices, assuming rational EV owners that minimize their costs.

Aggregator has to obtain earnings for the provided service, i.e. the average electricity price $\hat{\gamma}$ charged to EV owners should be greater or equal than the stochastic price $\lambda_{t,w}$, which is the spot price at the period time t in the scenario w. Thus, EV fleet operator has to manage the risk of energy trading by purchasing this commodity on the wholesale electricity market and selling it to users at a regulated price $\hat{\gamma}$. An example of this issue is described in (Borenstein, 2002).

In addition, the proposed solution needs a forecast of EV fleet response to the prices sent by an aggregator. For instance, Fig. 4.3 presents a sample of historical data related to the daily use in term of energy consumption for cars that are part of a fleet of identical EVs, such as those available from a car-sharing service in Italy, which can be utilized to determine and estimate driving patterns and define statistical models of energy requirement in urban areas. In particular, Fig. 4.3 shows a box plot for 10 EVs from January to April 2018. Note that the EV demand is greater during the night or early morning since the availability of public transport is reduced. Therefore, historical demand patterns can be used to forecast the driving behavior, then this fact provides information on the flexibility of EV demand.

4.2 Model formulation

In this section, an optimization problem for an EV aggregator is presented. A group of EVs is studied as a virtual battery within an EV fleet. This virtual battery represents the aggregated flexibility of EV owners and the charging curve can be modeled under this approach. Aggregator has information about the fleet, which can be estimated, for example, through the measurements of charging points, assuming that technology is available in the system. In addition, the fleet operator participates in the market as a utility/retailer in a price-taker approach.

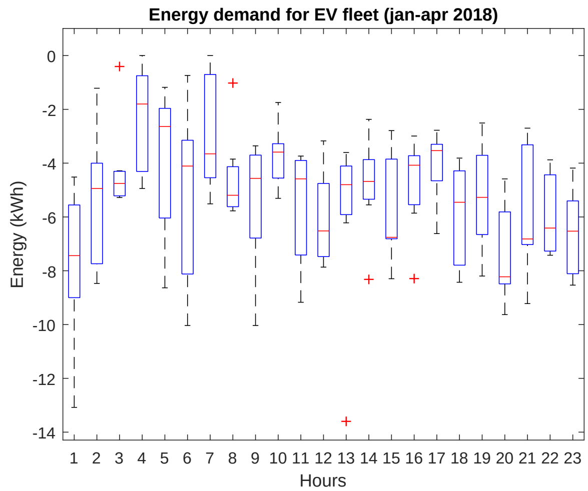

FIGURE 4.3: Energy demand for 10 EV from car-sharing service in Italy from January and April 2018.

This model proposes a solution to the aggregator planning problem by assuming rational behavior of EV owners grouped as a fleet. The problem is solved for a planning horizon of one day. In this work, time is discretized into 23 hour periods. Nevertheless, the duration of these time steps may be adjusted to attain a detailed modeling of the charging process. The proposed formulation is explained in four subsections: EV fleet model, the bilevel problem, transforming MPEC to MILP formulation, and finally, the resulting model.

4.2.1 Electric vehicle fleet model

Aggregator has to anticipate the EV behavior in order to design properly the retail price γ_t. In this sense, Aggregator assumes a model of EV fleet with uncertainty in its energy consumption. Particularly, EV aggregation is considered as a virtual battery with the inclusion of the forecasted energy demand (Baringo and Sánchez Amaro, 2017; Soares et al., 2016). The EV fleet model is formulated as follows:

$$E^S_{t,w} = E^S_{t-1,w} - E^T_{t,w} + \eta\delta P_t \;:\; \alpha_{t,w} \quad \forall t \tag{4.1a}$$
$$E^S_{t,w} \leq \overline{E^S_t} \;:\; \mu^a_{t,w} \quad \forall t \tag{4.1b}$$
$$\underline{E^S_t} \leq E^S_{t,w} \;:\; \mu^b_{t,w} \quad \forall t \tag{4.1c}$$
$$P_{t,w} \leq \overline{P_t} \;:\; \mu^c_{t,w} \quad \forall t \tag{4.1d}$$
$$P_{t,w}, E^S_{t,w} \geq 0 \quad \forall t \tag{4.1e}$$

Constraint (4.1a) is the energy balance for the virtual battery representing the EV fleet behavior, where $E^S_{t,w}$ is the stored energy, η is the charging efficiency, and P_t is the charging power requested by EV fleet. $E^T_{t,w}$ is a random variable that models the energy

demand of drivers, at each interval t, for using the EV fleet. w is a subindex that represents the uncertainties using a scenario approach. It accounts for the aggregated battery discharge produced by the use of the EVs during the interval of length δ. Constraints (4.1b), (4.1c) and (4.1d) are upper and lower stored energy bound ($\overline{E_t^S}$ and $\underline{E_t^S}$), and upper charging power limit $\overline{P_t}$, respectively. Lastly, restriction (4.1e) are the declarations of non-negative variables. Notice that $\alpha_{t,w}$, $\mu^a_{t,w}$, $\mu^b_{t,w}$, $\mu^c_{t,w}$ are the corresponding dual variables of constraints (4.1).
It is important to remain that the power and energy limits can be obtained by forecasting the driving patterns of EV users, added to the physical characteristics of the EV fleet and the charging stations. Then, this information can be acquired using historical data to predict the future driving requirements (see Fig. 4.3). This estimation process of parameters and stochastic variables is out of the scope of this work, and note that this input data is vital to obtain a suitable result in real applications of the proposed model. For instance, scenarios for the prices can be obtained using the method proposed in (Contreras et al., 2003).
Note that users decide their charging profile $P_{t,w}$ already knowing the retail price γ_t and its demand requirement (see decision sequence Fig. 4.2), i.e., consumers face the following problem:

$$\begin{aligned} P_{t,w} = \quad & \arg\min \quad && \sum_t \gamma_t P_t \\ & \text{subject to} \quad && \text{constraints (4.1)} \end{aligned} \tag{4.2}$$

Therefore, EV owners solve a deterministic optimization problem that is associated with their energy use and constraints. However, when designing the prices, Aggregator faces uncertainty in the demand. This situation is captured through a bilevel optimization problem which is explained in the following subsection.

4.2.2 Dynamic pricing model: a price-based solution

According to the contract, aggregator sends regulated prices to consumers in advance which are comprised between $\overline{\gamma}$ and $\underline{\gamma}$, and with a daily average of $\hat{\gamma}$. Furthermore, the fleet operator has to inform its bidding strategy in advance to the SO. With that information of all market participants, SO clears the market and then communicates to aggregator the market results. Given that the EV fleet operator has to design retail prices in advance, it faces uncertainty in spot prices and EV driving pattern.
A bilevel problem is posed to model the interaction between aggregator and EV fleet (see Fig. 4.2). In the upper-level, a profit-maximizing aggregator is considered to determine the prices γ_t. Lower-level is formulated as a constraint of the main problem, where EV fleet minimizes the charging costs. The stochastic optimization problem is

proposed below.

$$\begin{aligned} \underset{\Phi^U}{\text{maximize}} \quad & \mathbf{E}_w \left[\sum_t (\gamma_t - \lambda_{t,w}) P_{t,w} \right] \\ \text{subject to} \quad & \underline{\gamma} \leq \gamma_t \leq \overline{\gamma} \quad \forall t \\ & \frac{1}{nt} \sum_t \gamma_t = \hat{\gamma} \\ & P_{t,w} = \underset{\Phi^L}{\arg\min} \qquad \sum_t \gamma_t P_t \quad \forall w \\ & \qquad\qquad \text{subject to} \quad \text{constraints (4.1)} \end{aligned} \tag{4.3}$$

where $\Phi^U = \{\gamma_t\}$ and $\Phi^L = \{E^S_{t,w}, P_t\}$. Constraints of the upper-level ensure that the demand price is enclosed between $\underline{\gamma}$ and $\overline{\gamma}$, and also they enforce by contract that the dynamic price has a fixed daily average. Furthermore, $\lambda_{t,w}$ is the stochastic price given by utility to the aggregator and nt is total number of periods. Note that the Problem (4.3) has two random variables which are $\lambda_{t,w}$ and $E^T_{t,w}$.

4.2.3 Transforming into an MILP problem

The lower optimization problem, i.e. consumer decision, is changed by its Karush–Kuhn-Tucker (KKT) optimality conditions (Gabriel et al., 2013; Zugno et al., 2013). KKT formulation applies here since the lower-level problems are convex in the continuous variables $E^S_{t,w}$ and $P_{t,w}$, and since the upper-level variable, γ_t, can be considered as a parameter by the lower-level aggregation. In addition to the primal feasibility restrictions (4.1), the KKT necessary optimality conditions of lower-level problem hold the following:

$$\gamma_t - \eta\delta\alpha_{t,w} + \mu^c_{t,w} = 0 \quad \forall t, w \tag{4.4a}$$

$$\alpha_{t,w} - \alpha_{t+1,w} + \mu^a_{t,w} - \mu^b_{t,w} = 0 \quad \forall t < nt, w \tag{4.4b}$$

$$\alpha_{t,w} + \mu^a_{t,w} - \mu^b_{t,w} = 0 \quad \forall t = nt, w \tag{4.4c}$$

$$\mu^a_{t,w},\ \mu^b_{t,w},\ \mu^c_{t,w} \geq 0 \quad \forall t, w \tag{4.4d}$$

$$(E^S_{t,w} - \overline{E^S_t})\mu^a_{t,w} = 0 \quad \forall t, w \tag{4.4e}$$

$$(\underline{E^S_t} - E^S_{t,w})\mu^b_{t,w} = 0 \quad \forall t, w \tag{4.4f}$$

$$(P_{t,w} - \overline{P_t})\mu^c_{t,w} = 0 \quad \forall t, w \tag{4.4g}$$

Products of Lagrange multipliers and constrained continuous functions in the complementary slackness conditions, i.e, expressions (4.4e)-(4.4g), are equivalently replaced by linear equations through Fortuny-Amat transformation (Fortuny-Amat and McCarl, 1981), similar to (Zugno et al., 2013). Then, (4.4e)-(4.4g) can be substituted by following constraints.

$$
\begin{aligned}
-(E^S_{t,w} - \overline{E^S_t}) &\leq M(1 - z^a_{t,w}) \quad \forall t, w \\
\mu^a_{t,w} &\leq M z^a_{t,w} \quad \forall t, w \\
-(\underline{E^S_t} - E^S_{t,w}) &\leq M(1 - z^b_{t,w}) \quad \forall t, w \\
\mu^b_{t,w} &\leq M z^b_{t,w} \quad \forall t, w \\
-(P_{t,w} - \overline{P_t}) &\leq M(1 - z^c_{t,w}) \quad \forall t, w \\
\mu^c_{t,w} &\leq M z^c_{t,w} \quad \forall t, w \\
z^a_{t,w}, z^b_{t,w}, z^a_{t,w} &\in \{0, 1\}
\end{aligned} \tag{4.5}
$$

where M is a sufficiently large constant. This formulation introduces additional complexities by using binary variables $z^a_{t,w}$, $z^b_{t,w}$ y $z^c_{t,w}$, nevertheless, now all the restrictions are linear.

4.2.4 Relaxing bilinear term

The term $\gamma_t P_{t,w}$ is nonlinear then the strong duality theorem on the lower-level is employed in order to transform it into a linear expression. Therefore, the bilinear term can be stated as

$$
\begin{aligned}
\sum_t \gamma_t P_{t,w} = \sum_t [\alpha_{t,w} E^t_{t,w} - \mu^a_{t,w} \overline{E^S_t} + \mu^b_{t,w} \underline{E^S_t} - \mu^c_{t,w} \overline{P_t}] \\
- \alpha_{1,w} E^0_w
\end{aligned} \tag{4.6}
$$

where E^0_w is the initial condition of stored energy in the EV fleet. Expression (4.6) can be replaced in the objective function of problem (4.3).

4.2.5 Full MILP model

The equivalent single-level MILP formulation of the nonlinear MPEC problem is the following:

$$
\begin{aligned}
\underset{\Phi^{DP}}{\text{maximize}} \quad & \sum_w \pi(w) [\sum_t (\alpha_{t,w} E^t_{t,w} - \mu^a_{t,w} \overline{E^S_t} + \mu^b_{t,w} \underline{E^S_t} \\
& - \mu^c_{t,w} \overline{P_t} - \lambda_{t,w} P_{t,w})] - \sum_w \pi(w) \alpha_{1,w} E^0_w \\
\text{subject to} \quad & \underline{\gamma} \leq \gamma_t \leq \overline{\gamma} \quad \forall t \\
& \frac{1}{nt} \sum_t \gamma_t = \hat{\gamma} \\
& \text{constraints (4.1), (4.4}a\text{)} - \text{(4.4}d\text{), and (4.5).}
\end{aligned} \tag{4.7}
$$

being $\Phi^{DP} = \{E^S_{t,w}, P_{t,w}, \gamma_t, \alpha_{t,w}, \mu^a_{t,w}, \mu^b_{t,w}, \mu^c_{t,w}, z^a_{t,w}, z^b_{t,w}, z^c_{t,w}, X_{t,w}\}$. In addition, expected value in (4.3) is changed by the summation of $\pi(w)$, which means the probability of occurrence of each demand scenario w. A scenario tree can be used to capture the uncertainties in the model.

4.3 Numerical results

In this section, simulation results are presented to illustrate the proposed model to control an EV fleet demand by means of the price signal. Forecasted energy demand is taken from historical data of cars from a car-sharing service, which was collected during the Italian project Teinvein, corresponding to a fleet of small electric cars. The data are an average behavior extracted from several observations and represent aggregated information of 1000 vehicles. The simulation is performed for a planning horizon of 23 h divided into hourly time steps. Main characteristics of EV aggregation are provide in Table 4.1.

Number of EVs	1000
Capacity of each EV	12 kWh
Maximum charging power of each EV	3 kW
Charging efficiency	90%
Initial condition of EV fleet	4000 kWh

TABLE 4.1: Data of EV.

For car-sharing activities, it is expected that the most frequent trips are relatively short ones and the main activity is during the night, or weekend, when the availability of public transport is reduced. During workdays, the number of trips is shortened due to the fact that most of people are at work. In practice, hourly energy consumption is below 1 kWh for each vehicle. Fig 4.4 presents the forecasted energy demand for EV fleet in a day with the previous characteristics, which corresponds to the information of the last semester of 2017 from car-sharing service. Two scenarios with the same probability of occurrence are considered to evaluate the proposed model. A case with low energy consumption is referred as *scenario 1* and another with high demand is denoted by *scenario 2*, both cases are illustrated in Fig. 4.4.

For this simulation, power bounds of the virtual battery are set in fixed values to ease the analysis according to EV fleet conditions. Energy bounds were taken of aggregated state-of-charge from car-sharing service. However, those limits should be the results of studying the EV fleet behavior as a previous step of using the proposed model. The single-level mixed-integer linear programming problem (4.7) that results from the bilevel program (4.3) is solved using *FICO Xpress-Optimizer v29.01.10* under *Julia 0.6.2* on a Windows-based personal computer. Additionally, in order to evaluate the proposed model, another case with fixed price for all hours is considered, i.e, using the formulation (4.7) by replacing dynamic price constraints by a fixed price $\hat{\gamma}$, which is the average price agreed between the parties.

In this setting, fleet operator purchases the energy at certain prices in advance to the utility. For this example, the spot prices are considered as deterministic parameters and, based on duck curve and the work (Zugno et al., 2013). This price signal $\lambda_{t,w}$ is indicated in Fig. 4.5 as *utility*, which varies from €0.1/kWh to €0.39/kWh. For this simulation, aggregator sells energy to consumers 20% more expensive than that was purchased, i.e., $((1/nt)\sum_t \lambda_{t,w})1.2 = \hat{\gamma}$. Furthermore, the maximum and minimum prices for driver-owners are established as $\overline{\gamma} = \hat{\gamma} + \hat{\gamma}(0.3)$ and $\underline{\gamma} = \hat{\gamma} - \hat{\gamma}(0.3)$. For the above, aggregator problem is how to define retail prices to driver-owners given the uncertainties on driving patterns of this EV aggregation in order to maximize its profit.

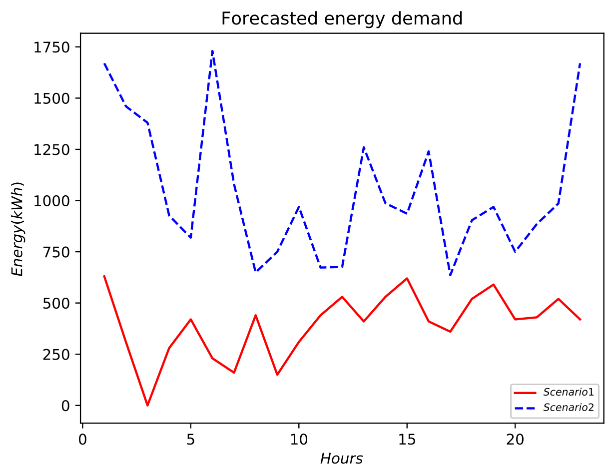

FIGURE 4.4: Forecasted energy demand.

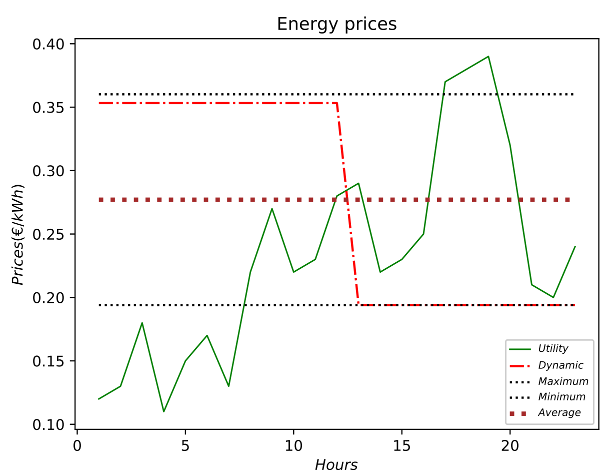

FIGURE 4.5: All energy prices.

Fig 4.5 presents the results of the proposed model in terms of prices for each case. The *average* price charged to consumers is €0.28/kWh considering a fixed pricing program. In the case of *Dynamic* pricing method, the price is most expensive in the first hours, between 1-12 hours, with a value of €0.35/kWh and for the rest of hours the best solution is to set a price of €0.19/kWh at intervals 13-23 hours.

Fig. 4.6 and 4.7 depict the charging power profile of EV fleet for each scenario given the prices obtained in the upper-level of problem (4.3). For *scenario 1* and *scenario 2*, the best charging curve under dynamic case is, in general, different between both pricing programs. An assumption of proposed formulation is that EV fleet at lower-level minimizes the operation cost, which means driver-owners are rational. Therefore, the charging profile depends on the prices given by aggregator and the demand requirement of EV trips.

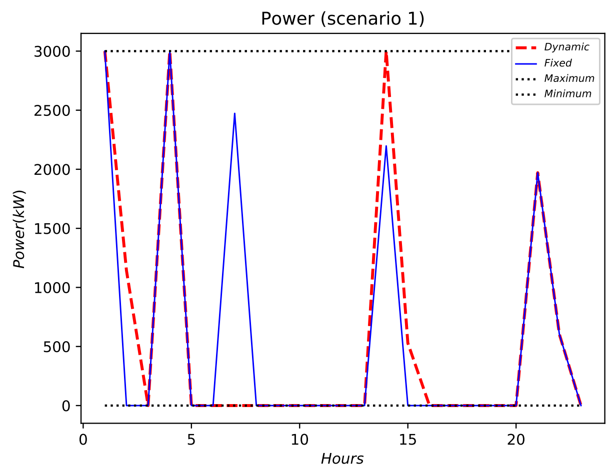

FIGURE 4.6: Scheduled power for *scenario 1*.

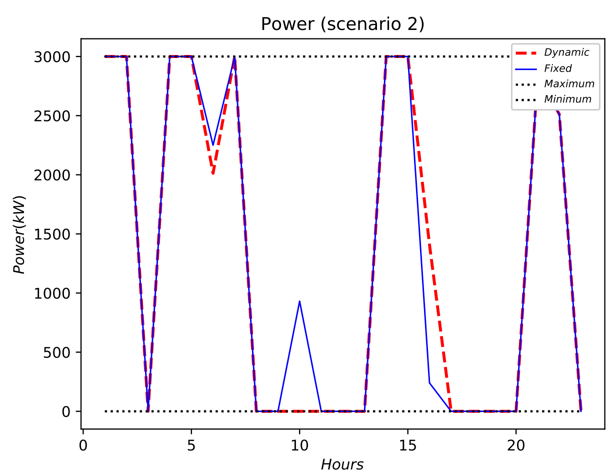

FIGURE 4.7: Scheduled power for *scenario 2*.

Fig. 4.8 and Fig. 4.9 present the results regarding stored energy in the EV fleet according to every scenario. For both cases, in dynamic pricing, tend to require less energy storage than the curve obtained in fixed price contract. For instance, Fig. 4.9 holds a similar behavior than the fixed program.

Expected profit for each pricing program is summarized in Table 4.2. Aggregator attains greater profit by employing a dynamic pricing contract than a fixed program as shown in Table 4.2. for this simulation, the proposed model represents a profit improvement of 2%. Nevertheless, this result can be increased by defining properly the agreed

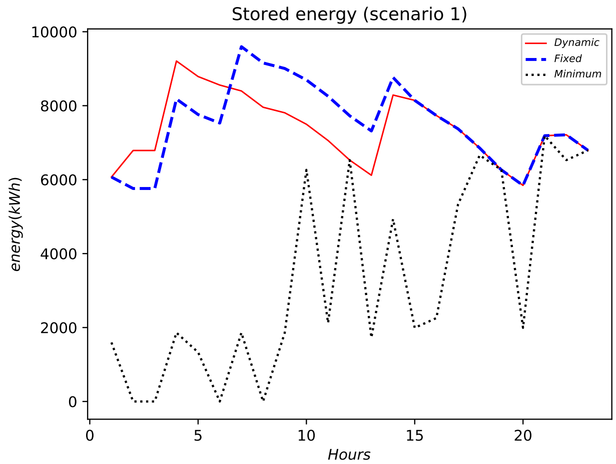

FIGURE 4.8: Stored energy of the EV fleet for *scenario 1*.

parameters ($\hat{\gamma}$, $\overline{\gamma}$ and $\underline{\gamma}$) in the contract. Hence, this method can be a useful tool to design prices, like TOU programs, for EV fleet operators which encourages the change of the energy consumption pattern of driver-owners by modifying retail price.

Program	Fixed price	Dynamic pricing
Expected profit	€2436.24	€2486.27

TABLE 4.2: Expected profit.

4.4 Remarks

In this Chapter was proposed a game theoretical model for EV fleet operator in order to design optimal prices that maximize its profit whereas driver-owners are minimizing costs as EV fleet. The problem was addressed using an MPEC programming algorithm since the relation between agents has hierarchical structure pertaining to the so-called Stackelberg (or leader-follower) games. The formulation allowed linking the EV fleet decisions and the aggregator objectives to determine the price signal and the optimal load pattern. As results, in the dynamic-price case, the EV fleet operator, while maximizing its profits, sends EV owners a price-incentive to shift their electric charging demand to periods where the aggregator can get better benefits than a fixed price contract.

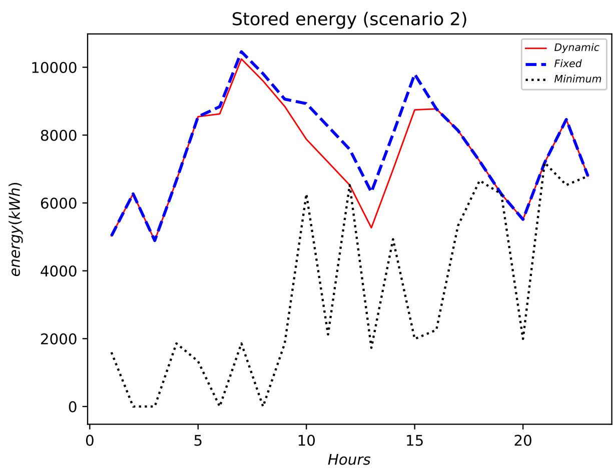

FIGURE 4.9: Stored energy of the EV fleet for *scenario 2*.

Chapter 5

Incentive-based demand response model for Cournot competition

This Chapter presents an analysis of competition between generators when incentive-based demand response is employed in an electricity market. Thermal and hydropower generation are considered in the model. Some related works were presented in Subsection 1.1.4. A smooth inverse demand function is proposed using a sigmoid and two linear functions for modeling the consumer preferences under incentive-based demand response program. Generators compete to sell energy bilaterally to consumers. The profit of each agent is posed as an optimization problem, then the competition result is found by solving simultaneously KKT conditions for all generators. A Nash-Cournot equilibrium is found when the system operates normally and at peak demand times when DR is required. For this model, DR is activated when the demand side exceed a defined threshold a priori by SO in order to guarantee some objectives of a smart grid. Then, a novel demand curve is proposed in order to understand the effect of electricity market behavior when an incentive-based DR program, like PTR, is held to diminish the energy consumption at the peak periods.
The agents involved in an electricity market in competition with DR are shown in Fig. 5.1. SO is responsible for arbitrage services in order to establish a proper environment for competition and gaming. The generators have different technologies, costs, revenues, and each firm seeks to maximize its profit (the difference between producers' revenue and costs). Furthermore, the aggregators carry out the request to users of reducing energy consumption, namely, DR process. The main goal is to estimate the equilibrium price under gaming environment. This competition is less than perfect, some firms are able to influence the market price through their actions. Such optimization problems set up which is called in game theory a non-cooperative game (Vega Redondo, 2003; Gabriel et al., 2013; Tirole, 1988; Osborne, 1995; Varian, 1992). The solution of such a game is called a Nash equilibrium and represents a market equilibrium under imperfect competition. A game among generators with different technologies in an electricity market is studied if DR is required when the demand side exceed a defined threshold a priori by SO in order to guarantee some objectives of a smart grid. This threshold is determined from all customer baseline load and desired energy reduction during peak times. These findings have been published in (Vuelvas and Ruiz, 2018).

The key points are summarized as follows:

- A novel incentive-based DR model is proposed. Demand curve is formulated by using a sigmoid function between two linear polynomials to depict the energy threshold when DR is required. This formulation is a continuous function with

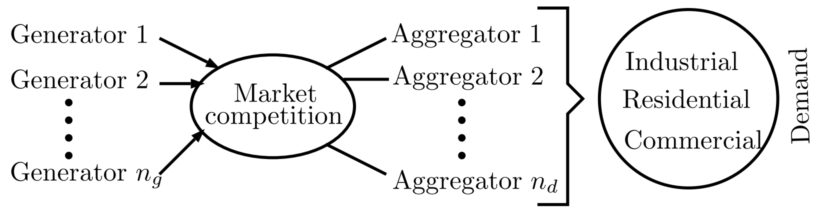

FIGURE 5.1: Gaming in the electricity market. Where n_g and n_d are the total number of generators and aggregators in the electricity market, respectively.

finite marginal value in the demand curve. In particular, it is an alternative modeling of DR to (Su and Kirschen, 2009), where, the demand curve has two parts: perfect inelastic behavior and price responsive consumers. The inconvenience of (Su and Kirschen, 2009) is that demand does no have perfect inelastic role since the consumers have a limited willingness to pay.

- A Nash-Cournot equilibrium is formulated as a complementary problem in the presence of DR (Gabriel et al., 2013). The generators compete without a centralized program. Cournot gaming is compared when an electricity market operates normally and when an incentive-based DR is active during peak times.

The Chapter is organized as follows. Section 5.1 introduces the agent models in an electricity market. Section 5.2, the problem formulation as Cournot Competition in the presence of DR is developed. Numerical results are described in Section 5.3. Discussion is presented in Section 5.4. Finally, some final comments are drawn in Section 5.5.

5.1 Agent models

In this section, a novel demand model is proposed for studying an incentive-based DR program within an electricity market. This formulation illustrates the wholesale market behavior during a day. In addition, generator models are posed under Cournot Competition.

5.1.1 Demand response model

The most important decision unit of microeconomic theory is the demand (Varian, 1992; Mas-Colell, Whinston, and Green, 1995). Then, a new approach for modeling the demand is posed when consumers participate in an incentive-based DR program. This part holds similar setting from Chapters 2 and 3, then, this information is added so that each chapter can be read independently. Let $T = \{1, 2, ..., n_t\}$ be the set of periods to take into account in the horizon time, where n_t is last hour, that is, $n_t = 24$. An aggregated demand is considered for this DR rebate model. The decision-maker's preferences are specified by giving smooth utility function $G(q_t)$, where q_t is the energy consumption at time t. $G(q_t)$ depicts the level of satisfaction obtained by the demand as a function of the total power consumption.The utility curve satisfies the following properties as proposed in (Chen et al., 2012; Samadi et al., 2012; Fahrioglu and Alvarado, 2000; Vega Redondo, 2003; Osborne, 1995):

Property 1: $G(q_t)$ is assumed as a concave function with respect to q_t. This implies that the marginal benefit of users is a nonincreasing function.

$$\frac{d^2G(q_t)}{dq_t^2} \leq 0 \quad \forall t \in T$$

Property 2: The marginal benefit is nonnegative.

$$\frac{dG\left(q_t\right)}{dq_t} \geq 0 \quad \forall t \in T$$

Property 3: $G(q_t)$ is zero when the consumption level is zero.

$$G\left(0\right) = 0 \quad \forall t \in T$$

The market price is p_t^* at the time t. The superscript star indicates the equilibrium price. For each generator, the cost function is assumed increasing with respect to the total energy production capacity. In addition, the cost function is strictly convex. Then, other definitions are considered as follows.

Definition 1. *The demand energy total cost is* $\pi(q_t) = p_t^* q_t$.

Definition 2. $G(q_t)$ *is approximated by a second order polynomial around* $\overline{q}_t$, $\quad \forall t \in T$. *In general, a quadratic function is considered.*

$$G\left(q_t\right) = -\frac{\gamma_t}{2}\left(q_t - \overline{q}_t\right)^2 + p_t^*\left(q_t - \overline{q}_t\right) + k \quad \forall t \in T$$

being $k = \overline{q}_t\left(\frac{\gamma_t}{2}\overline{q}_t + p_t^*\right)$ *a constant value, obtained by Property 3.*

Definition 3. *The payoff function is defined as* $U_t\left(q_t\right) = G(q_t) - \pi(q_t)$, *which indicates the user benefit of consuming* q_t *energy during the interval* t.

Basically, incentive-based DR programs request customers for curtailing demand in response to a price signal or economic incentive. Typically the invitation to reduce demand is made for a specific time period or peak event. There are some concepts in order to define DR rebate program:

Definition 4. *Baseline* (β_t)*: the amount of energy the user would have consumed in the absence of a request to reduce (counterfactual model) (Deng et al., 2015). This quantity can not be measured, then this is estimated from the previous consumption of the agent. In this work, the aggregated baseline corresponds to the sum of all customer baseline loads in order to propose a DR threshold required in the electricity market.*

q_t is the actual use, namely, the amount of energy that aggregated demand actually consumes during the event period.

Definition 5. *Load Reduction* ($\triangle_t\left(\beta_t, q_t\right)$)*: the difference between the baseline and the actual use.*

$$\beta_t - q_t = \triangle_t$$

In incentive-based DR programs, the rebate is only received if there is an energy reduction. Otherwise, the user does not get any incentive or penalty. Mathematically,

Definition 6. *Let p_2 be the rebate price received by the demand due to energy reduction in peak periods. The DR incentive π_2 is given by,*

$$\pi_2\left(q_t; p_{2t}, \beta_t\right) = \begin{cases} p_{2t}\triangle_t = p_{2t}(\beta_t - q_t) & q_t < \beta_t \\ 0 & q_t \geq \beta_t \end{cases} \quad \forall t \in T$$

Next, the demand payoff function with DR rebate program is written as:

$$\hat{U}_t\left(q_t; p_{2t}, \beta_t\right) = G(q_t) - \pi(q_t) + \pi_2(q_t; p_{2t}, \beta_t) \quad \forall t \in T \tag{5.1}$$

In this Chapter, the inverse demand function is formulated to develop the Cournot's model of oligopoly. The inverse demand function is given by $p_t\left(q_t\right) = \frac{dU_t(q_t)}{dq_t}$. Where $p_t\left(q_t\right)$ is the price function at the time t.

Accordingly, the inverse demand function without DR is obtained from Definition 3. Next, the linear inverse demand function is derived as follows.

$$p_t\left(q_t\right) = -\gamma_t q_t + \gamma_t \overline{q_t} \quad \forall t \in T \tag{5.2}$$

Whether the demand payoff function with DR is considered when $q_t < \beta_t$, then the inverse demand function is given by,

$$\widetilde{p_t}\left(q_t\right) = -\gamma_t q_t + \left(\gamma_t \overline{q_t} - p_{2t}\right) \quad \forall t \in T \tag{5.3}$$

In order to model the electricity market with DR during peak hours, a sigmoid function between both inverse demand functions (Eq. (5.2) and Eq. (5.3)) is proposed. Fig. 5.2 (a) depicts a novel demand function that models incentive-based DR at market level. The novel inverse demand function is presented as follows.

$$\hat{p}_t\left(q_t\right) = -\gamma_t q_t + \left(\gamma_t \overline{q_t} - \frac{p_{2t}}{1 + e^{\alpha(-q_t + \xi)}}\right) \quad \forall t \in T \tag{5.4}$$

where α is a constant value which represents the smoothness of the sigmoid function that joins the two straight lines and ξ is the threshold level to perform the DR process. Notice that this demand model represents a preference alteration of consumers. Fig 5.2 (b) shows the case when the supply curve is intersected by demand curve for $q_t > \beta_t$. The equilibrium price $\hat{p}_t{}^*$ is less than the energy price given by the inverse demand function $p_t\left(q_t\right)$. In addition, the energy consumption decreases to $\hat{q}_t{}^*$ owing to the incentive price π_2, which is requested by setting the threshold ξ. The incentive is paid to consumers when the DR program is required. Besides, SO determines the threshold ξ according to the available energy (water reservoirs, fuels, etc.), the estimated baseline β_t and the energy consumption patterns. Then aggregators encourage customers for carrying out the energy reduction.

In (Su and Kirschen, 2009), the demand model has two parts: consumers that have perfect inelastic behavior, they are represented by an infinite marginal value; and users that participate in a DR program, they can place bid price with a finite marginal value in the demand curve. However, the drawback of the proposal (Su and Kirschen, 2009) is that demand does not have perfect inelastic behavior because the consumers have a limit willingness for energy payment. In this work, demand always has a finite marginal

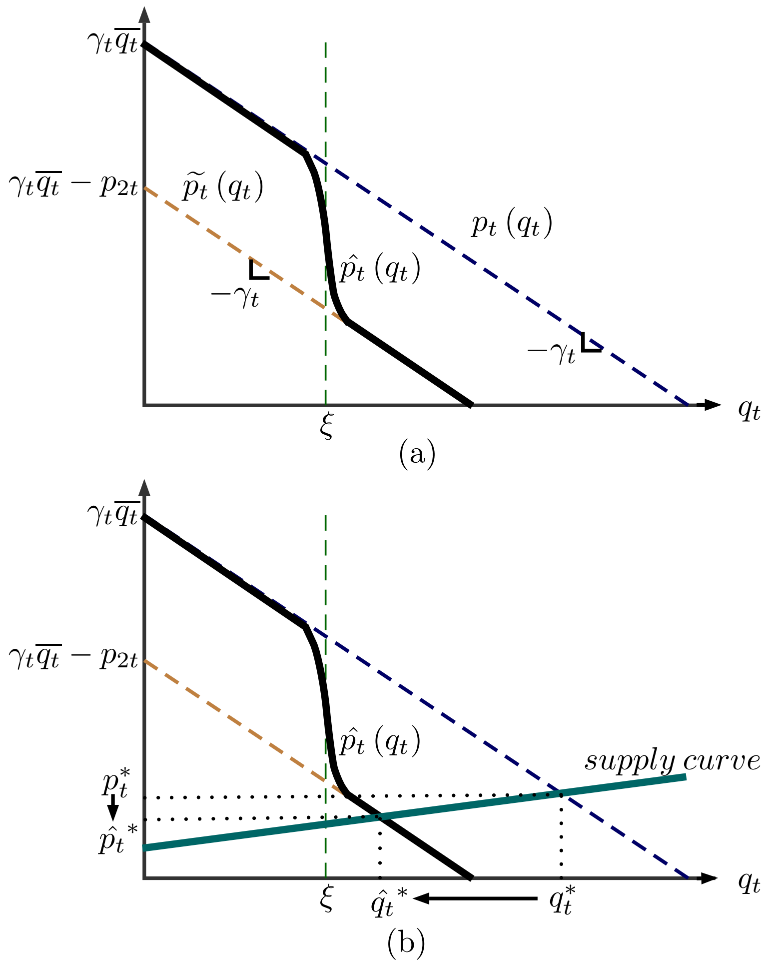

FIGURE 5.2: Inverse demand function.

value, hence, this model is an alternative to represent the DR behavior in an electricity market.

5.1.2 Supply model

The relationship between total energy from all generators and price lead to make the supply curve. In this work, producers try to anticipate the results of their actions on the price, then the market experiences imperfect competition. SO has arbitrage services, commands DR threshold, and manages the transmission assets as its functions into the electricity market. Therefore, each generator seeks independently to maximize its own economic benefits. The profit is given by its revenue from sold energy minus the cost of generating it. Two kinds of power suppliers are considered: thermal generators are represented with quadratic costs and hydropower are formulated with fixed costs (Genc and Thille, 2008).

Thermal generation modeling

The thermal cost is given by an increasing quadratic function. Let r_{ta} be the power generated by producer $a \in A$ at the period t, where A is the set of thermal generators. Thus, the costs have the following form: $c1_a r_{ta} + \frac{c2_a}{2} r_{ta}^2 + c3_a$, being $c1_a$, $c2_a$ and $c3_a$ constant values that depict private information. These quadratic costs are stated because thermal power has an expensive economic behavior (Genc and Thille, 2008). In this sense, each generator uses its knowledge of the inverse demand function ($p_t(q_t)$ or $\hat{p}_t(q_t)$) to anticipate its own effect on the market price in order to maximize its profit. Then, the optimization problem for thermal generator is posed as follows.

$$\begin{aligned} \max \ & \textstyle\sum_{t \in T} \left[p_t(q_t)\, r_{ta} - \left(c1_a r_{ta} + \frac{c2_a}{2} r_{ta}^2 + c3_a \right) \right] \\ \text{s.t.} \ & r_{ta} \le r_a^+ \ : \ \mu_{ta}^T \quad \forall t \in T \\ & r_{ta} \ge 0 \quad \forall t \in T \end{aligned} \tag{5.5}$$

where r_a^+ is the maximum value of the energy that each thermal power can generate in each period. μ_{ta}^T is dual variable for the first constraint. This model does not consider ramp restriction, minimum uptime and downtime, among other constraints.

Hydropower modeling

Hydropower is included in competition into the electricity market. The hydro generator has a production function $H_{tb}(w_{tb})$ which represents the conversion of water release to energy, where w_{tb} is the water discharge of hydro reservoir for each generator $b \in B$. For this kind of producer, a fixed cost $c4_b$ is formulated. Hence, the optimization problem is to maximize the profits by each hydropower.

$$\begin{aligned} \max \ & \textstyle\sum_{t \in T} \left[p_t(q_t)\, H_{tb}(w_{tb}) - c4_b \right] \\ \text{s.t.} \ & w_{tb} \le w_{tb}^+ \ : \ \mu_{tb}^H \, \forall t \in T \\ & w_{tb} \ge 0 \, \forall t \in T \end{aligned} \tag{5.6}$$

where w_{tb}^+ is the maximum value of the water release at the time t for the generator b. μ_{tb}^H is dual variable for the first constraint.

5.2 Incentive-based demand response in Cournot competition

A Cournot competition is developed for studying the proposed DR model that is described in section 2. This model assumes that generators cannot collude or form a cartel, and they seek to maximize their own profit based on demand model. This section describes the game between market participants in order to settle the energy price by solving simultaneously the optimization problems (5.5) and (5.6), as presented in (Gabriel et al., 2013). Now, the definition of Nash equilibrium is stated as follows.

Definition 7. *Considering the game* $G = \left\langle I, \{S_i\}_{i=1,2,..n_i}, \{\psi_i\}_{i=1,2,...,n_i} \right\rangle$*, where* I *is the players set,* S_i *is the strategies set of each player and* $\psi_i : \prod_{i \in I} S_i \to \mathbb{R}$ *is the utility function of each generator.* $(s_1^*, ...s_i^*)$ *is a Nash equilibrium whether* $\forall i \in I$ *player is true that:* $\psi_i\left(s_i^*, s_{-i}^*\right) \ge \psi_i\left(s_i, s_{-i}^*\right)$*,* $\forall s_i \in S_i$*, being* s_{-i} *all strategies except the player* i *(Vega Redondo, 2003).*

Remark 1. *Nash equilibrium has two interpretations: s_i^* is the best response to s_{-i}^* or it does not exist unilateral incentives to deviate from Nash equilibrium. Furthermore, an equilibrium problem can be solved using KKT conditions of several interrelated optimization problem (Gabriel et al., 2013).*

First, the aim is to solve Eq. (5.5) and Eq. (5.6) in the case when no DR is required, i.e, using the demand model $p_t(q_t)$ given by Eq. (5.2). Next, the situation when DR is requested, Eq. (5.4) is used as demand model given by the threshold defined by SO. In order to find the solution, the KKT conditions of each agent are solved simultaneously. In particular, if DR is applied then the demand side shifts its energy requirement during the day in order to maintain its preferences and satisfaction levels, therefore, balance constraints are included in this case to model this behavior, namely, $\sum_{t \in T} r_t + H_t = D_n$ is added to the optimization problem, being D_n the estimated net demand without DR.

In this work, a duopoly is assumed for understanding the effect of the proposed DR model. In particular, two generators are employed to find the Nash-Cournot equilibrium: one thermal energy producer and one hydropower according to the suggested supply curve from Section 5.1. For simplicity, the subscript a and b from Eq. (5.5) and Eq. (5.6) are removed because there is one generator per technology. Therefore, the net energy consumption is $q_t = r_t + H_t(w_t)$ for each $t \in T$. The KKT conditions with and without DR are presented as follows.

5.2.1 Electricity market without demand response

First, considering the case when DR is not required in the market. The KKT conditions are rewritten as complementary model by using Eq. (5.2) which are shown below.

$$\begin{gathered} 0 \le r_t\left(2\gamma_t + c_2\right) + \left(\gamma_t H_t\left(w_t\right) + c_1\right) - \gamma_t \overline{q_t} + \mu_t^T \perp r_t \ge 0 \quad \forall t \in T \\ 0 \le \mu_t^T \perp r^+ - r_t \ge 0 \quad \forall t \in T \end{gathered} \tag{5.7}$$

$$\begin{gathered} 0 \le \frac{dH_t(w_t)}{dw_t}\left[\gamma_t r_t + 2\gamma_t H_t\left(w_t\right) - \gamma_t \overline{q_t}\right] + \mu_t^H \perp w_t \ge 0 \quad \forall t \in T \\ 0 \le \mu_t^H \perp w_t^+ - w_t \ge 0 \quad \forall t \in T \end{gathered} \tag{5.8}$$

where (5.7) and (5.8) are the resulting conditions for thermal generation and hydropower, respectively. Note that Eq. (5.7) and Eq. (5.8) do not have interconnected periods since each hour of a day has energy consumption requirement which is depicted by an independent demand model.

5.2.2 Electricity market with demand response

Next, the KKT conditions are presented as follows when the market has an incentive command by reducing energy consumption given by the demand model from Eq. (5.4).

$$\begin{gathered} 0 \le l_r + p_{2t}\frac{e^{\alpha(r_t + H_t(w_t))}\left[e^{\alpha(r_t + H_t(w_t))} + e^{\alpha\xi}(\alpha r_t + 1)\right]}{\left[e^{\alpha(r_t + H_t(w_t))} + e^{\alpha\xi}\right]^2} + r_t\left(2\gamma_t + c_2\right) \\ + \left(\gamma_t H_t\left(w_t\right) + c_1\right) - \gamma_t \overline{q_t} + \mu_t^T \perp r_t \ge 0 \quad \forall t \in T \\ 0 \le \mu_t^T \perp r^+ - r_t \ge 0 \quad \forall t \in T \\ \textstyle\sum_{t \in T} r_t + H_t = D_n, \quad l_r \text{ free} \end{gathered} \tag{5.9}$$

$$
\begin{aligned}
0 \le l_h + \tfrac{dH_t(w_t)}{dw_t}\left[p_{2t}\frac{e^{\alpha(r_t+H_t(w_t))}\left[e^{\alpha(r_t+H_t(w_t))}+e^{\alpha\xi}(\alpha H_t(w_t)+1)\right]}{\left[e^{\alpha(r_t+H_t(w_t))}+e^{\alpha\xi}\right]^2}\right] \\
+\tfrac{dH_t(w_t)}{dw_t}\left[\gamma_t r_t + 2\gamma_t H_t(w_t) - \gamma_t \overline{q_t}\right] + \mu_t^H \perp w_t \ge 0 \quad \forall t \in T \\
0 \le \mu_t^H \perp w_t^+ - w_t \ge 0 \quad \forall t \in T \\
\textstyle\sum_{t\in T} r_t + H_t = D_n, \quad l_h \text{ free}
\end{aligned} \tag{5.10}
$$

where (5.9) and (5.10) are the KKT conditions for thermal generation and hydropower if DR is applied, respectively. l_r and l_h are the dual variables associated to balance constraints for thermal generation and hydropower, correspondingly. For this case, a balance constraint between all periods is added to model the shift in energy load that consumers perform to maintain their activities or the comfort levels during a day.

5.3 Numerical results

The analysis of numerical examples involves three aspects: the effect of demand response, the study of consumer and generator surplus and the effect on the incentive variation. The simulation is performed in *GAMS 24.7.4* using *PATH* as the solver.
In Table 5.1, the simulation parameters are shown in order to illustrate the new approach of demand model with DR given by Eq. (5.4). The simulation data are based on (Forouzandehmehr, Han, and Zheng, 2014; Genc and Thille, 2008; Cunningham, Baldick, and Baughman, 2002).

$T = \{1,2,3,...,24\}\ h$	$\xi = 1000\ MWh,\ H_t(w_t) = w_t,\ \alpha = 0.1$
$c1 = 10\,\$$	$\gamma_t = \$\{0.065, 0.067, 0.063, 0.063, 0.06, 0.065, 0.062, 0.068, 0.065, 0.067, 0.063, 0.067, 0.068, 0.069, 0.062, 0.061, 0.067, 0.067, 0.055, 0.054, 0.055, 0.065, 0.063, 0.061\}\ /MWh^2$
$c2 = 0.025\,\$$	$\gamma_t\overline{q_t} = \$\{92.4, 93.82, 95.67, 99.2, 95.32, 94.56, 90.56, 91.14, 90.19, 92.23, 91.45, 95.7, 104.45, 103.13, 101.54, 91.87, 103.95, 95.23, 120.19, 120.35, 120.23, 108.4, 95.67, 95.67\}\ MWh/$
$c3 = 0\,\$$	$p_{2t} = \$\{0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0, 0,0,20,20,20,0,0,0\}\ /MWh$
$c4 = 0\,\$$	$r^+ = 500\ MWh,\ w_t^+ = 1000\ \frac{acre-ft}{h}\ \forall t \in T$

TABLE 5.1: Simulation parameters.

First, the Cournot competition between generators without DR is shown. Fig. 5.3 depicts the results of gaming between thermoelectric and hydroelectric when the inverse demand function is given by Eq. (5.2). The equilibrium energy versus hours in a day are depicted in Fig. 5.3 according to the generator technology and the total electrical energy delivered to customers. Simulations are made in a 24-hour horizon. Hydropower has the main participation in the market due to it does not have the variable cost, therefore, it is cheaper than the thermal generation. The peak time occurs between 19 to 21 hours. Lastly, the net demand for all periods without DR is $D_n = 25476.4\ MWh$.

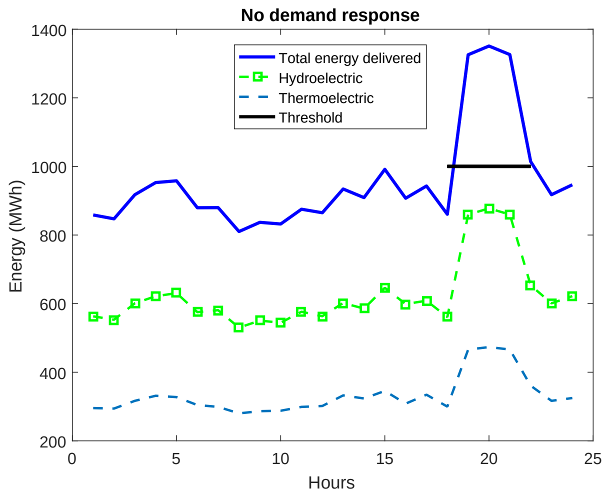

FIGURE 5.3: Cournot competition without demand response.

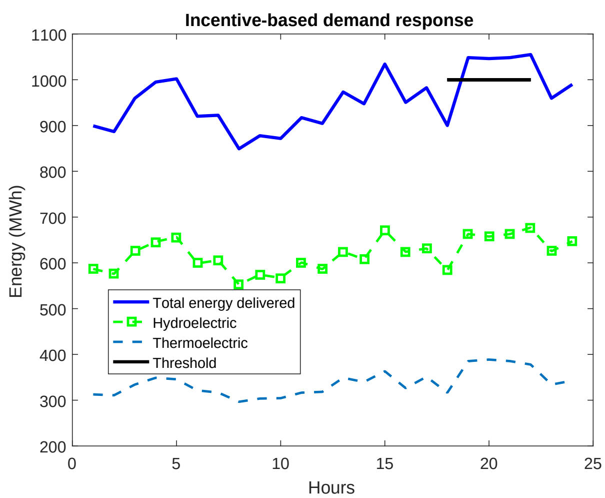

FIGURE 5.4: Cournot competition with demand response.

Next, Fig. 5.4 presents the competition case when there is an incentive command if the energy consumption is greater than the threshold $\xi = 1000\ MWh$. Below this value, the DR benefits do not apply. For instance, notice that the total energy delivered at the hour 20 is about 1351 MWh in Fig. 5.3, i.e., above the baseline. As long as, in Fig. 5.4, the energy value at the same time is around 1046 MWh, therefore, the energy reduction is approximately 305 MWh since the demand behavior is altered by the incentive payment given by the definition 6. In addition, the reduction proportion is similar for each technology. For this case, $D_n = 25476.4\ MWh$ is used as estimated demand to solve Eq (5.9) and Eq. (5.10). Hence, the net demand is the same for both situations. If DR is employed, consumers shift energy consumption to an off period in order to maintain their satisfaction level.

5.3.1 The effect of demand response

In Fig. 5.5, the effect of DR in terms of energy is shown. Whether the consumption is higher than the threshold value ($\xi = 1000\ MWh$) and if the period has reduction incentive then the DR model stimulates the consumers to reduce the energy consumption patterns. This behavior is found because the economic incentive p_{2t} is introduced on the inverse demand function. Thus, this incentive payment can be understood as an alteration of consumer preferences made by SO, to alleviate the system in contingency situations where an energy reduction is required in the grid operation. For this example, the cutback during peak times is about 21.5%. This percentage changes according to the threshold selected by SO. Furthermore, demand shifts the energy requirement to other periods to hold the same activities during the day by increasing energy consumption at off-peak times.

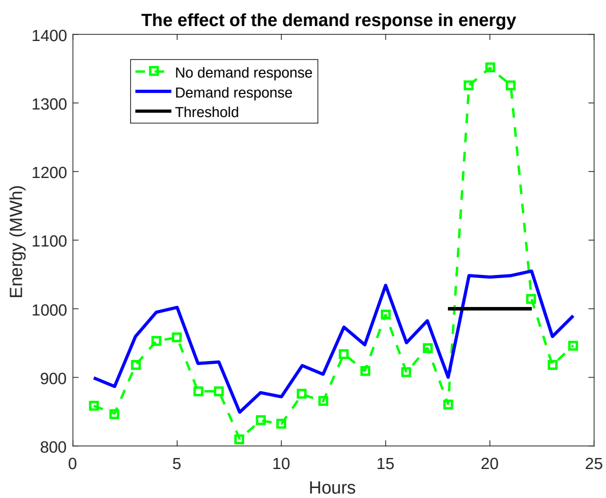

FIGURE 5.5: The effect of the demand response in energy.

In Fig. 5.6, the effects of the DR in term of prices is depicted. The fashion in which a consumer reduces his energy is through economic stimulus or incentives. Under this program, consumers are rewarded by a reduction of load in peak hours. Fig. 5.6 shows that DR reduces the market prices since obeying the law of supply and demand for all periods. However, for obtaining the energy reduction, SO must pay an economic incentive in order to motivate the load curtailment by consumers. Therefore, in certain events, the incentive-based DR is a reasonable alternative to overcome contingency scenarios in the electric power system. At these times, it is more cost-effective to diminish demand than to increase supply or induce power outages to maintain the balance in the electrical grid.

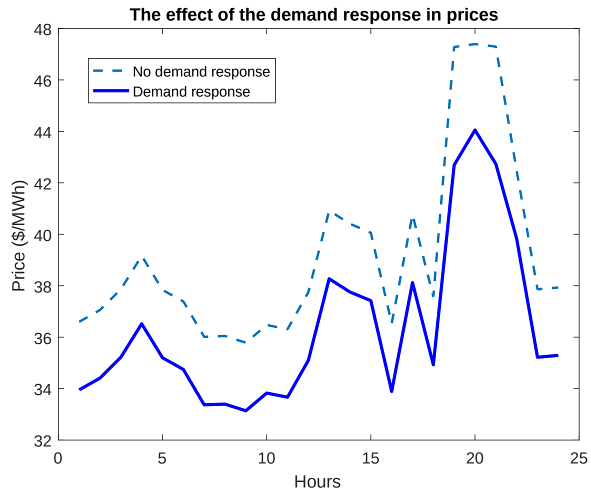

FIGURE 5.6: The effect of demand response in prices.

5.3.2 Consumer and producer surplus

Consumer and generator surplus are shown in Fig. 5.7. The producer surplus is calculated from the objective functions (Eq. (5.5) and Eq. (5.8)). Whereas the consumer surplus is obtained by replacing directly the inverse demand function (5.4) in $\int_0^{q_t} p_t\left(E'\right) dE' - p_t^* q_t$. An important feature of the incentive-based DR program is that the generators decrease their profit or surplus when DR is required. This effect is due to the reduction performed by users, in which, the prices are affected by the inverse demand curve stated when the energy exceeds the threshold. Moreover, consumers are rewarded by a reduction in their energy bill whether they reduce their consumption. Therefore, users have a greater economic surplus with DR program than not participating, taking into account the previous definitions for this model.

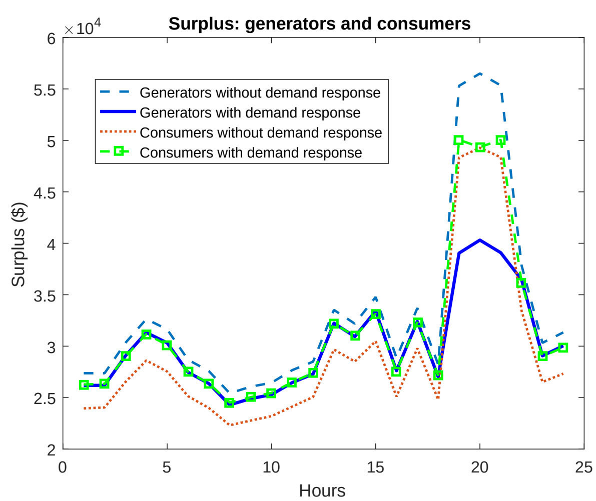

FIGURE 5.7: Consumer and producer surplus.

In Fig. 5.8, the generator surplus by technology is illustrated. Hydropower has the major participation in the electricity market, therefore, it suffers the greatest reduction in its benefits. Whereas that thermoelectric reduces slightly its profit. In general, the energy reduction depends on the participation of each generator in the energy market. For instance, 30.4% and 23.8% are the reduction percentages of hydroelectric and thermoelectric at 20 hour, respectively.

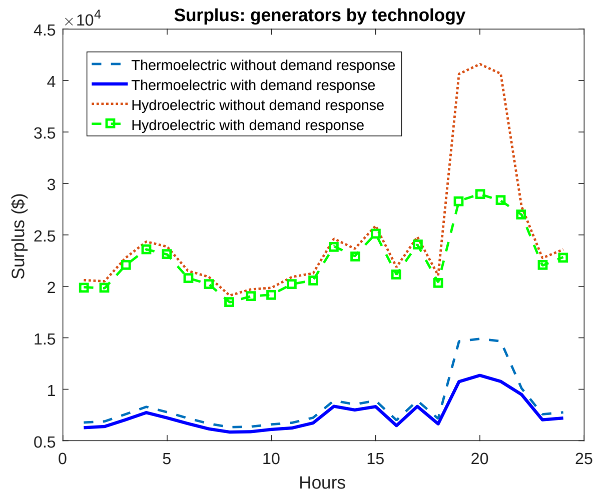

FIGURE 5.8: Generators surplus by technology.

5.3.3 Incentive effect in demand response

For analyzing the incentive effect in this kind of DR program, the simulation parameters are set in $\gamma_t = \$0.054 \,/\, MWh^2$ and $\gamma_t \overline{q_t} = \$120.35 \,/\, MWh$ for one period. Next, the aim is to change the incentive price in order to understand what happens to the energy cutback, market price, and participant surplus. In Fig. 5.9, the percentage energy reduction and the market price are shown according to the incentive. The immediate effect of DR is to decrease the electric power requirement and, by the law of supply and demand, the market price declines as increases the incentive signal. For instance, whether the incentive price p_{2t} is equal to \$10 / MWh, then the market price is \$43.67 / MWh and the energy reduction is 8.6%.

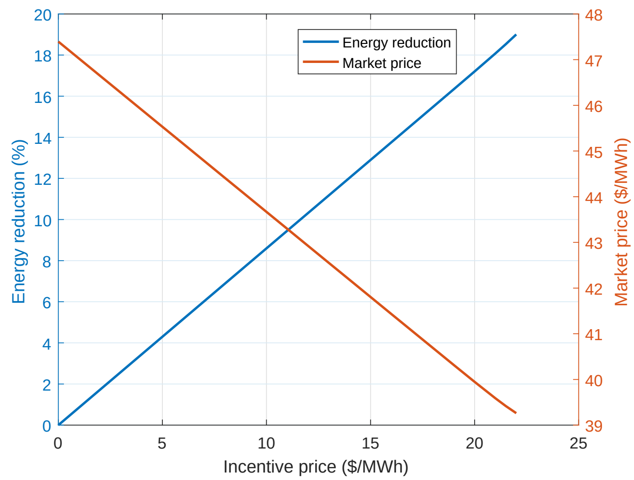

FIGURE 5.9: Energy reduction and market price affected by the DR incentive.

Fig. 5.10 illustrates the generation and demand surplus as increases the incentive price. The amount of energy to be dispatch is less when DR is required. Then, the generation profit diminishes also caused by decreasing market prices. Therefore, the main achievement of this incentive-based DR program is to guarantee the power availability in peak events or to provide a solution to a contingency situation, e.g., low water levels in reservoirs of hydroelectric power. Furthermore, consumers perceive more economic benefits when they are participating in the DR program since the net price is cheaper whether they reduce their consumption. For example, if the incentive price is $\$10/MWh$ then users notice an increase about 3.8% of their surplus, while, the generation has a decrease around 16.1% of the profit.

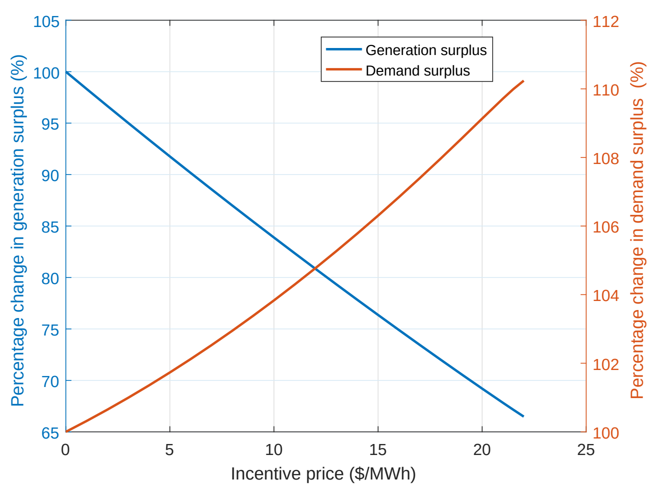

FIGURE 5.10: Generation and demand surplus behavior according to the incentive price

5.4 Discussion

Incentive-based DR is a program where consumers receive incentive payment according to a counterfactual model, namely, they are rewarded by a price multiplied by energy reduction which is measured from household baselines. Therefore, this quantity is estimated by SO from previous energy consumption. During peak times, SO evaluates the demand forecast (baseline), power availability, transmission constraints, costs of power outages, among others in order to define if DR procedure is required.
The proposed model allows a SO to know the market behavior in an imperfect environment to take decisions when an indirect DR method is employed. This model includes a threshold which can be interpreted as a guide or reference value of the expected energy reduction during peak time at market level. In addition, the threshold can be derived or estimated from the subset of consumers that are willing to participate in DR. This information can be collected by aggregators and analyzed by SO.
Economic policy is focused on how to define the incentive signal to determine a trade-off between agent surplus, grid constraints, and market objectives. For instance, under this model, the following question could be assessed: where does the incentive come from? This could be addressed by adding a fixed cost to the market price so that it can be used to encourage consumers to reduce their energy consumption in a contingency situation. Moreover, most of the literature is concentrated on directly studying DR at distribution side without considering all effects in the system as a whole. Therefore, this model provides tools to determine choices on indirect DR methods in electricity markets.
This new price responsive demand structure is an economic tool for analyzing DR programs in imperfect markets. In addition, this approach can be extended to study the operation of centralized systems which could result in the following benefits: price responsive demand can make the power market more competitive during peak times; it also can improve the predictability of demand requirements and could provide rapid response to emergency shortage conditions; finally, it can postpone the need for generation investment and delay the need for certain transmission upgrades by decelerating the growth in peak demand.

5.5 Findings

In this Chapter was developed an analysis of Cournot competition in an incentive-based DR program. A new demand curve was proposed for modeling consumer preferences in order to include DR in the electricity market. Incentives for consumers were considered as the DR program. The demand model was devised as a composition of two linear functions and a sigmoid, which represents an energy threshold for analyzing the load reduction in this kind of DR programs. It was found that the incentive-based DR is a cost-effective solution to reduce energy consumption during peak times. However, this program affects negatively the generator surplus under competition environment.
The proposed model can be employed to study price responsive demand in wholesale electricity markets where consumers have the opportunity to reduce voluntary their consumption according to incentive signals. Particularly, price response characteristic can enable development of enhanced operational systems to take advantage of the predictable behavior of short term consumption patterns that are associated with wholesale

price conditions. For instance, the model can work as decision-making tool for grid operators to defer more expensive dispatch options and reduce transmission congestion costs.

Chapter 6

Conclusions

In this dissertation, new approaches have been proposed to analyze and integrate DR solutions in smart grids. This work has covered the topics related to indirect DR management from consumers to market level, including EV applications. The achieved contributions can be applied to evaluate the performance of current incentive-based DR programs and to serve as a foundation for the implementation of innovative agreements that improve the DR operation in smart grids as a vital resource to contribute in reducing carbon emissions.

Most of the literature is focused on describing and reporting the advantages and disadvantages of incentive-based DR program. In the first part of this work was rigorously proposed a model to deeply understand the rational decisions of consumers when they participate in this kind of DR contract. Consumer's decision problem with choices under uncertainty was modeled using stochastic optimization techniques to quantify the gaming behavior, measuring the impact of baseline alteration from participant users. Therefore, this new insight can allow devising better agreements or policies between aggregator or utility and end-users in order to obtain a real energy reduction.

A feature studied in Chapter 2 is that consumers have the certainty that they are called to participate in DR. In Chapter 3 was presented a contract where users have uncertainty in being required to perform an energy reduction. This agreement was denoted by the concept "probability of call" and it proposes a straightforward solution to limit the gaming opportunities on the baseline. In this contract was demonstrated the voluntary participation and asymptotic truthfulness, which are two desirable properties in a contract. In addition, an important advantage of this agreement is susceptibility to be implemented since the proposed solution uses information in terms of energy which can be obtained by power monitors. Therefore, this contract can be an alternative to the traditional PTR program.

Another DR agreement result was developed for EV applications. An interaction model between a fleet operator and EV owners was presented to serve as price planner to improve the battery charging profile. A bilevel problem was proposed to depict the decision-making structure among involved agents, since its hierarchical framework and the number of time periods make the problem into a dynamic game. A state-space model for EV fleet was considered to account the demand dynamics. This approach has been formulated as an alternative to model consumer behavior avoiding solutions that choose demand elasticities or utility functions. Price-based DR programs can be addressed under this formulation to derive prices that encourage certain aggregated charging profile for EV fleet.

Another contribution of this dissertation was to analyze generator competition in electricity markets under DR framework. A smooth inverse demand function was proposed using a sigmoid and two linear functions for modeling the consumer preferences under incentive-based demand response program. This model can be used as a decision-making tool for grid operators to study the incentive signal in indirect DR management by examining the trade-off between agent surplus, grid constraints, and market objectives.

There are still many aspects to be explored about indirect demand response management in smart grids. For instance, a vital question is the determination of the number of participant users and the clustering in the contract based on the probability of call in order to reasonably achieve the objectives of the electrical grid. Likewise, an interesting topic is the extension of the mentioned contract using the assumptions and definitions of behavioral economics, extending the notion of the rational consumers in DR framework. As well, In Chapter 4, the forecasting process of parameters and demand for EVs represents an important previous step to determine properly energy prices that induce certain behavior in the EV fleet. Finally, experimental validations of the proposed models would be important contributions as the next step of this research.

Bibliography

Aghaei, Jamshid and Mohammad Iman Alizadeh (2013). "Demand response in smart electricity grids equipped with renewable energy sources: A review". In: *Renewable and Sustainable Energy Reviews* 18, pp. 64–72. ISSN: 13640321. DOI: 10.1016/j.rser.2012.09.019.

Aghajani, Saemeh and Mohsen Kalantar (2017). "Operational scheduling of electric vehicles parking lot integrated with renewable generation based on bilevel programming approach". In: *Energy* 139, pp. 422–432. ISSN: 03605442. DOI: 10.1016/j.energy.2017.08.004. URL: https://doi.org/10.1016/j.energy.2017.08.004.

Albadi, M. H. and E. F. El-Saadany (2008). "A summary of demand response in electricity markets". In: *Electric Power Systems Research* 78.11, pp. 1989–1996. ISSN: 03787796. DOI: 10.1016/j.epsr.2008.04.002.

Allcott, Hunt (2012). "Real-Time Pricing and Electricity Market Design". In: *NYU Working Paper. Ney York University (NYU)*.

Antunes, Pedro, Pedro Faria, and Zita Vale (2013). "Consumers performance evaluation of the participation in demand response programs using baseline methods". In: *2013 IEEE Grenoble Conference* 2011, pp. 1–6. DOI: 10.1109/PTC.2013.6652420.

Baringo, Luis and Raquel Sánchez Amaro (2017). "A stochastic robust optimization approach for the bidding strategy of an electric vehicle aggregator". In: *Electric Power Systems Research* 146, pp. 362–370. ISSN: 03787796. DOI: 10.1016/j.epsr.2017.02.004.

Bizzarri, F. et al. (2016). "Electric vehicles state of charge and spatial distribution forecasting: A high-resolution model". In: *IECON Proceedings (Industrial Electronics Conference)*, pp. 3942–3947. DOI: 10.1109/IECON.2016.7794060.

Bloustein, E (2005). "Assessment of customer response to real time pricing". In: *Rutgers-The State University of New Jersey, Tech. Rep*, pp. 1–23.

Borenstein, Severin (2002). "The Trouble With Electricity Markets: Understanding California's Restructuring Disaster". In: *Journal of Economic Perspectives* 16.1, pp. 191–211. ISSN: 0895-3309. DOI: 10.1257/0895330027175. URL: http://pubs.aeaweb.org/doi/10.1257/0895330027175.

Chao, Hung (2011). "Demand response in wholesale electricity markets: the choice of customer baseline". In: *Journal of Regulatory Economics* 39.1, pp. 68–88. ISSN: 0922-680X. DOI: 10.1007/s11149-010-9135-y.

Chen, Yan et al. (2012). "A cheat-proof game theoretic demand response scheme for smart grids". In: *IEEE International Conference on Communications*, pp. 3362–3366. ISSN: 15503607. DOI: 10.1109/ICC.2012.6364397.

Contreras, Javier et al. (2003). "ARIMA models to predict next-day electricity prices". In: *IEEE Transactions on Power Systems* 18.3, pp. 1014–1020. ISSN: 08858950. DOI: 10.1109/TPWRS.2002.804943. arXiv: 1707.03541.

Cunningham, Lance B., Ross Baldick, and Martin L Baughman (2002). "An Empirical Study of Applied Game Theory: Transmission Constrained Cournout Behaviour". In: *IEEE Transactions on power systems* 17.1, pp. 166–172.

Datchanamoorthy, S. et al. (2011). "Optimal time of use pricing for residential load control". In: *2011 IEEE International Conference on Smart Grid Communications (SmartGridComm)*, pp. 375–380. DOI: 10.1109/SmartGridComm.2011.6102350.

Deng, Ruilong et al. (2015). "A Survey on Demand Response in Smart Grids: Mathematical Models and Approaches". In: *IEEE Transactions on Industrial Informatics* 11.3, pp. 1–1. ISSN: 1551-3203. DOI: 10.1109/TII.2015.2414719.

Diaz, C., F. Ruiz, and D. Patino (2017). "Modeling and control of water booster pressure systems as flexible loads for demand response". In: *Applied Energy* 204, pp. 106–116. ISSN: 03062619. DOI: 10.1016/j.apenergy.2017.06.094. URL: http://dx.doi.org/10.1016/j.apenergy.2017.06.094.

Dobakhshari, Donya Ghavidel and Vijay Gupta (2016). "A Contract Design Approach for Phantom Demand Response". In: *ArXiv e-prints* 1611.09788.November 2016, pp. 1–16. arXiv: 1611.09788. URL: http://adsabs.harvard.edu/abs/2016arXiv161109788G.

Dobakshari, D G and V Gupta (2016). "Optimal contract design for incentive-based demand response". In: *American Control Conference*, pp. 3219–3224.

Ericson, Torgeir (2009). "Direct load control of residential water heaters". In: *Energy Policy* 37.9, pp. 3502–3512. ISSN: 03014215. DOI: 10.1016/j.enpol.2009.03.063.

Fahrioglu, Murat and Fernando L. Alvarado (2000). "Designing incentive compatible contracts for effective demand management". In: *IEEE Transactions on Power Systems* 15.4, pp. 1255–1260. ISSN: 08858950. DOI: 10.1109/59.898098.

— (2001). "Using utility information to calibrate customer demand management behavior models". In: *IEEE Transactions on Power Systems* 16.2, pp. 317–323. ISSN: 08858950. DOI: 10.1109/59.918305.

Faria, Pedro, Zita Vale, and Pedro Antunes (2013). "Determining the adjustment baseline parameters to define an accurate customer baseline load". In: *IEEE Power and Energy Society General Meeting* 2011. ISSN: 19449925. DOI: 10.1109/PESMG.2013.6672749.

Faruqui, Ahmad et al. (2011). *Will Smart Prices Induce Smart Charging of Electric Vehicles?* DOI: 10.2139/ssrn.1915658. URL: http://www.ssrn.com/abstract=1915658.

Forouzandehmehr, Najmeh, Zhu Han, and Rong Zheng (2014). "Stochastic Dynamic Game between Hydropower Plant and Thermal Power Plant in Smart Grid Networks". In: *IEEE Systems Journal* 10.1, pp. 88–96. ISSN: 19328184. DOI: 10.1109/JSYST.2014.2317555.

Fortuny-Amat, José and Bruce McCarl (1981). "A representation and economic interpretation of a two-level programming problem". In: *Journal of the Operational Research Society*. ISSN: 14769360. DOI: 10.1057/jors.1981.156.

Gabriel, Steven A. et al. (2013). *Complementarity Modeling in Energy Markets*. Vol. 1. Springer New York, p. 636. ISBN: 978-1-4419 -6122-8. DOI: 10.1007/978-1-4419-6123-5.

Garcia, Alfredo, Enrique Campos-Nañez, and James Reitzes (2005). "Dynamic Pricing and Learning in Electricity Markets". In: *Operations Research* 53.2, pp. 231–241. ISSN: 0030-364X. DOI: 10.1287/opre.1040.0159.

Genc, Talat S and Henry Thille (2008). *Dynamic Competition in Electricity Markets : Hydropower and Thermal Generation*.

Gonzalez Vaya, Marina and Goran Andersson (2015). "Optimal bidding strategy of a plug-in electric vehicle aggregator in day-ahead electricity markets". In: *IEEE Transactions on Power Systems* 30.5, pp. 1–6. DOI: 10.1109/EEM.2013.6607304.

Goudarzi, Hadi, Safar Hatami, and Massoud Pedram (2011). "Demand-side load scheduling incentivized by dynamic energy prices". In: *2011 IEEE International Conference on Smart Grid Communications, SmartGridComm 2011*, pp. 351–356. DOI: 10.1109/SmartGridComm.2011.6102346.

Gruosso, G. (2017a). "Analysis of impact of electrical vehicle charging on low voltage power grid". In: *2016 International Conference on Electrical Systems for Aircraft, Railway, Ship Propulsion and Road Vehicles and International Transportation Electrification Conference, ESARS-ITEC 2016*. DOI: 10.1109/ESARS-ITEC.2016.7841365.

Gruosso, Giambattista (2017b). "Model Based Approach for evaluation of Energy Consumption of Electrical Vehicles". In: *Vehicle Power and Propulsion Conference (VPPC), 2017 IEEE*, pp. 1–6. ISBN: 9781538613177.

Hatami, S. and M. Pedram (2010). "Minimizing the Electricity Bill of Cooperative Users under a Quasi-Dynamic Pricing Model". In: *Smart Grid Communications (SmartGridComm), 2010 First IEEE International Conference on*, pp. 421–426. DOI: 10.1109/SMARTGRID.2010.5622080.

Herter, Karen (2007). "Residential implementation of critical-peak pricing of electricity". In: *Energy Policy* 35.4, pp. 2121–2130. ISSN: 03014215. DOI: 10.1016/j.enpol.2006.06.019.

Hu, Junjie et al. (2016). "Electric vehicle fleet management in smart grids: A review of services, optimization and control aspects". In: *Renewable and Sustainable Energy Reviews* 56, pp. 1207–1226. ISSN: 18790690. DOI: 10.1016/j.rser.2015.12.014.

Huang, Shaojun et al. (2015). "Distribution Locational Marginal Pricing Through Quadratic Programming for Congestion Management in Distribution Networks". In: *IEEE Transactions on Power Systems* 30.4, pp. 2170–2178. ISSN: 08858950. DOI: 10.1109/TPWRS.2014.2359977.

Khajavi, P., H. Abniki, and A. B. Arani (2011). "The role of incentive based demand response programs in smart grid". In: *2011 10th International Conference on Environment and Electrical Engineering, EEEIC.EU 2011 - Conference Proceedings*. DOI: 10.1109/EEEIC.2011.5874702.

LLP, Troutman Sanders (2013). *FERC Settles Investigation Concerning Demand Response Products in PJM*. URL: http://www.troutmansandersenergyreport.com/2013/06/ferc-settles-investigation-concerning-demand-response-products-in-pjm/ (visited on 01/27/2016).

Ma, Hongyao, David C Parkes, and Valentin Robu (2017). "Generalizing Demand Response Through Reward Bidding". In: *Proceedings of the 16th International Conference on Autonomous Agents and Multiagent Systems.*

Ma, Z J, D S Callaway, and I A Hiskens (2013). "Decentralized Charging Control of Large Populations of Plug-in Electric Vehicles". In: *Ieee Transactions on Control Systems Technology* 21.1, pp. 67–78. ISSN: 0743-1546. DOI: Doi10.1109/Tcst.2011.2174059.

Madaeni, Seyed Hossein and Ramteen Sioshansi (2013). "The impacts of stochastic programming and demand response on wind integration". In: *Energy Systems* 4.2, pp. 109–124. ISSN: 18683967. DOI: 10.1007/s12667-012-0068-7.

Mas-Colell, A, M D Whinston, and J R Green (1995). *Microeconomic Theory*. Oxford student edition. Oxford University Press. ISBN: 9780195073409.

Mathieu, Johanna L, Duncan S Callaway, and Sila Kiliccote (2011). “Examining uncertainty in demand response baseline models and variability in automated responses to dynamic pricing”. In: *Proceedings of the IEEE Conference on Decision and Control*, pp. 4332–4339. ISSN: 01912216. DOI: 10.1109/CDC.2011.6160628.

Meir, Reshef, Hongyao Ma, and Valentin Robu (2017). “Contract Design for Energy Demand Response”. In: *ArXiv e-prints*. arXiv: 1705.07300. URL: http://adsabs.harvard.edu/abs/2017arXiv170507300M.

Mohagheghi, Salman and Neda Raji (2015). “Dynamic demand response : A solution for improved energy efficiency for industrial customers”. In: *IEEE Industry Applications Magazine* 21.2, pp. 54–62. ISSN: 10772618. DOI: 10.1109/MIAS.2014.2345799.

Mohajeryami, Saeed, Milad Doostan, and Peter Schwarz (2016). “The impact of Customer Baseline Load (CBL) calculation methods on Peak Time Rebate program offered to residential customers”. In: *Electric Power Systems Research* 137, pp. 59–65. ISSN: 03787796. DOI: 10.1016/j.epsr.2016.03.050.

Muthirayan, Deepan et al. (2016). “Mechanism Design for Self-Reporting Baselines in Demand Response”. In: *American Control Conference (ACC), 2016*, pp. 1446–1451. ISBN: 9781467386814.

— (2017). “Mechanism Design for Demand Response Programs”. In: pp. 1–17. arXiv: 1712.07742. URL: http://arxiv.org/abs/1712.07742.

Newsham, Guy R. and Brent G. Bowker (2010). “The effect of utility time-varying pricing and load control strategies on residential summer peak electricity use: A review”. In: *Energy Policy* 38.7, pp. 3289–3296. ISSN: 03014215. DOI: 10.1016/j.enpol.2010.01.027.

Osborne, Martin J (1995). *A course in game theory*. First. London: MIT press, p. 115. ISBN: 0262650401. DOI: 10.1016/0898-1221(95)90101-9.

Palensky, Peter and Dietmar Dietrich (2011). “Demand side management: Demand response, intelligent energy systems, and smart loads”. In: *IEEE Transactions on Industrial Informatics* 7.3, pp. 381–388. ISSN: 15513203. DOI: 10.1109/TII.2011.2158841.

Rajagopal, Ram et al. (2013). “Risk-limiting dispatch for integrating renewable power”. In: *International Journal of Electrical Power and Energy Systems* 44.1, pp. 615–628. ISSN: 01420615. DOI: 10.1016/j.ijepes.2012.07.048.

Roughgarden, Tim (2016). *Lectures Notes on Algorithmic Game Theory*. Cambridge University Press, p. 352. ISBN: 9781107172661.

Samadi, P et al. (2012). “Advanced Demand Side Management for the Future Smart Grid Using Mechanism Design”. In: *Smart Grid, IEEE Transactions on* 3.3, pp. 1170–1180. ISSN: 1949-3053. DOI: 10.1109/TSG.2012.2203341.

Sarker, Mushfiqur R., Yury Dvorkin, and Miguel A. Ortega-Vazquez (2016). “Optimal participation of an electric vehicle aggregator in day-ahead energy and reserve markets”. In: *IEEE Transactions on Power Systems* 31.5, pp. 3506–3515. ISSN: 08858950. DOI: 10.1109/TPWRS.2015.2496551.

Severin Borenstein (2014). *Peak-Time Rebates: Money for Nothing?* URL: http://www.greentechmedia.com/articles/read/Peak-Time-Rebates-Money-for-Nothing (visited on 01/27/2016).

Shao, Shengnan et al. (2010). “Impact of TOU rates on distribution load shapes in a smart grid with PHEV penetration”. In: *2010 IEEE PES Transmission and Distribution Conference and Exposition: Smart Solutions for a Changing World*, pp. 1–6. ISSN: 2160-8555. DOI: 10.1109/TDC.2010.5484336.

Siano, Pierluigi (2014). "Demand response and smart grids - A survey". In: *Renewable and Sustainable Energy Reviews* 30, pp. 461–478. ISSN: 13640321. DOI: 10.1016/j.rser.2013.10.022.

Soares, Joao et al. (2016). "A Stochastic Model for Energy Resources Management in Smart Grids". In: *Electric Power System Research* 143, pp. 599–610.

Su, Chua Liang and Daniel Kirschen (2009). "Quantifying the Effect of Demand Response on Electricity Markets". In: *IEEE Transactions on Power Systems* 24.3, pp. 1199–1207. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2009.2023259.

Tirole, Jean (1988). *The Theory of Industrial Organization*. MIT press, p. 479. ISBN: 9780262200714.

Tyagi, Rajesh and Jason W. Black (2010). "Emergency demand response for distribution system contingencies". In: *2010 IEEE PES Transmission and Distribution Conference and Exposition: Smart Solutions for a Changing World*, pp. 1–4. ISBN: 9781424465477. DOI: 10.1109/TDC.2010.5484598.

Varaiya, P, F Wu, and J Bialek (2011). "Smart Operation of Smart Grid: Risk-Limiting Dispatch". In: *Proceedings of the IEEE* 99.1, pp. 40–57. ISSN: 0018-9219. DOI: 10.1109/JPROC.2010.2080250.

Vardakas, John S, Nizar Zorba, and Christos V Verikoukis (2015). "A Survey on Demand Response Programs in Smart Grids: Pricing Methods and Optimization Algorithms". In: *IEEE Communications Surveys & Tutorials* 17.1, pp. 152–178.

Varian, Hall (1992). *Microeconomics Analysis*. 3rd ed. Norton & Company, p. 506. ISBN: 0393957357.

Vaya, Marina Gonzalez and Goran Andersson (2012). "Centralized and decentralized approaches to smart charging of plug-in Vehicles". In: *IEEE Power and Energy Society General Meeting*, pp. 1–8. ISSN: 19449925. DOI: 10.1109/PESGM.2012.6344902.

Vega Redondo, Fernando (2003). *Economics and the theory of Games*. Cambridge University Press, p. 528.

Villar, J and H Rudnick (2003). "Hydrothermal market simulator using game theory: Assessment of market power". In: *Ieee Transactions on Power Systems* 18.1, pp. 91–98. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2002.807061.

Von Neumann, J and O Morgenstern (1944). *Theory of Games and Economic Behavior*. Princeton University Press, p. 625. ISBN: 0691003629. DOI: 10.1177/1468795X06065810.

Vuelvas, José and Fredy Ruiz (2015). "Demand response: Understanding the rational behavior of consumers in a Peak Time Rebate Program". In: *in Automatic Control (CCAC), 2015 IEEE 2nd Colombian Conference on*, pp. 1–6. DOI: 10.1109/CCAC.2015.7345196.

— (2017). "Rational consumer decisions in a peak time rebate program". In: *Electric Power Systems Research* 143, pp. 533–543. ISSN: 03787796. DOI: 10.1016/j.epsr.2016.11.001.

— (2018). "A novel incentive-based demand response model for Cournot competition in electricity markets". In: *Energy Systems*. ISSN: 1868-3975. DOI: 10.1007/s12667-018-0271-2. URL: https://doi.org/10.1007/s12667-018-0271-2.

Vuelvas, José, Fredy Ruiz, and Giambattista Gruosso (2018a). "A Contract for Demand Response based on Probability of Call". In: *2018 IEEE International Conference on Industrial Technology (ICIT)*, pp. 1095–1100. ISBN: 9781509059485.

— (2018b). "Limiting gaming opportunities on incentive-based demand response programs". In: *Applied Energy* 225, pp. 668–681. ISSN: 03062619. DOI: 10.1016/j.apenergy.2018.05.050. arXiv: 1802.08604.

Wijaya, Tri Kurniawan, Matteo Vasirani, and Karl Aberer (2014). "When Bias Matters: An Economic Assessment of Demand Response Baselines for Residential Customers". In: *IEEE Transactions on Smart Grid* 5.4, pp. 1755–1763. ISSN: 1949-3053. DOI: 10.1109/TSG.2014.2309053.

Wolak, Frank A (2006). "Residential Customer Response to Real-Time Pricing: The Anaheim Critical-Peak Pricing Experiment". In: *Center for the Study of Energy Markets. UC Berkeley: Center for the Study of Energy Markets*.

Wu, Hongyu et al. (2016). "A Game Theoretic Approach to Risk-Based Optimal Bidding Strategies for Electric Vehicle Aggregators in Electricity Markets Markets With Variable Wind Energy Resources". In: *IEEE Transactions on Sustainable Energy* 7.1, pp. 374–385. DOI: 10.1109/TSTE.2015.2498200.

Yao, Jian, Ilan Adler, and Shmuel S. Oren (2008). "Modeling and Computing Two-Settlement Oligopolistic Equilibrium in a Congested Electricity Network". In: *Operations Research* 56.1, pp. 34–47. ISSN: 0030-364X. DOI: 10.1287/opre.1070.0416. URL: http://pubsonline.informs.org/doi/abs/10.1287/opre.1070.0416.

Yao, Jian, Shmuel S. Oren, and Ilan Adler (2007). "Two-settlement electricity markets with price caps and Cournot generation firms". In: *European Journal of Operational Research* 181.3, pp. 1279–1296. ISSN: 03772217. DOI: 10.1016/j.ejor.2005.10.073.

Yoon, Sung Guk et al. (2016). "Stackelberg-Game-Based Demand Response for At-Home Electric Vehicle Charging". In: *IEEE Transactions on Vehicular Technology* 65.6, pp. 4172–4184. ISSN: 00189545. DOI: 10.1109/TVT.2015.2440471.

Yu, Rongshan, Wenxian Yang, and Susanto Rahardja (2012). "A statistical demand-price model with its application in optimal real-time price". In: *IEEE Transactions on Smart Grid* 3.4, pp. 1734–1742. ISSN: 19493053. DOI: 10.1109/TSG.2012.2217400.

Zhou, Datong P., Munther A. Dahleh, and Claire J. Tomlin (2017). "Eliciting Private User Information for Residential Demand Response". In: *ArXiv e-prints*. arXiv: 1703.00972. URL: http://adsabs.harvard.edu/abs/2017arXiv170300972Z.

Zhou, Yue et al. (2017). "Optimal scheduling of aggregated thermostatically controlled loads with renewable generation in the intraday electricity market". In: *Applied Energy* 188, pp. 456–465. ISSN: 03062619. DOI: 10.1016/j.apenergy.2016.12.008.

Zhu, Quanyan, Peter Sauer, and Tamer Basar (2013). "Value of demand response in the smart grid". In: *2013 IEEE Power and Energy Conference at Illinois, PECI 2013*, pp. 76–82. ISBN: 9781467356022. DOI: 10.1109/PECI.2013.6506038.

Zugno, Marco et al. (2013). "A bilevel model for electricity retailers' participation in a demand response market environment". In: *Energy Economics* 36, pp. 182–197. ISSN: 01409883. DOI: 10.1016/j.eneco.2012.12.010.