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Marginal effects on the portfolio's Value at Risk: a VaR - Unconditional Quantile Regression approach

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Abstract

This article proposes and evaluates an econometric model that aims to clarify the relationship between Value at Risk and various factors that may affect it. This model, called VaR-UQR (VaR - Unconditional Quantile Regression), estimates the relationship between VaR and various risk factors and offers an extension to the technique of unconditional quantile regression estimation (UQR) proposed by Firpo et. al (2009). The UQR model differs from the conditional quantile regression (QR) model regarding that the estimators found through QR must be interpreted as the effect of a change in the explanatory variable on the quantile of the explained variable, conditional on a certain value of the explanatory variable, while those found through UQR can be interpreted as the effect on the unconditional quantiles of the explained variable. Once the tests of the proposed model were performed, it was found that the Value at Risk of international reserves portfolios is exposed to changes in the risk-free interest rate. Additionally, when analyzing the movements coming from the financial and banking sector the VaR of this type of portfolios is covered.

JEL classification: C14, C22, E44, G11, G21.

Keywords: Unconditional quantile regression; RIF regression; CAViaR; Value at Risk; contagion effect; risk management.

1. Introduction

Among the participants of the financial system there are also entities responsible for the economic and financial stability, such as governments[1], central banks and supranational entities. These entities invest liquidity excesses in portfolios, commonly known as international[2] reserves portfolios (data used for the estimations made in this work), to maintain the value of money in time, while ensuring that their resources remain available to attend possible events of economic or financial turmoil. As participants of the financial system these institutions are exposed to both systemic and idiosyncratic risk through the investments in their portfolios, so it is important to evaluate the type of risks to which they are exposed to. Some examples of these institutions are the Inter-American Development Bank, the Latin American Reserve Fund, the Central Bank of Brazil, and the Banco de la República of Colombia. This kind of entities allow their investment portfolios to hold bonds issued by corporations, with ratings up to A-, according to the S&P scale, and in different currencies, usually from developed countries[3].

After the financial crisis of 2008, regulation was created about which entities should monitor the financial system and how it should be monitored. An example of this was the creation of the Financial Stability Oversight Council (FSOC) which, in order to fulfil its mandates, must start by correctly estimating systemic risk, as argued by Bisias et. al. (2012).

When systemic risk events occur, these organisms must use their liquidity excesses to bail out institutions that play an important role in the financial and/or economic system and that may increase the depth of the risk event. Therefore, their portfolios must be covered or behave in a counter-cyclical manner when these events take place.

Usually, these institutions maintain a conservative and rigorous risk management in their reserve portfolios, which may include restrictions in the type of assets they can be exposed to or limits for their risk metrics. Based on these guidelines, an asset allocation process is usually carried out. Within the industry standards for asset allocation is found the modern portfolio theory, which proposes methods to optimize portfolios, that is, to minimize portfolio risk given a level of return, or to maximize return given a certain level of risk. Markowitz (1956) was one of the pioneers in proposing his model of portfolio optimization based on the mean and variance of returns. However, more risk measures have been considered when efficiently selecting the assets in a portfolio. Some of these measures are "Value at Risk" (VaR) and "Tacking Error" (TE), among others.

VaR is widely used for reporting regulatory metrics to control bodies, and for this reason it must be constantly monitored. Additionally, it is one of the most widely used risk measures given its simplicity and comparability. It is defined as the maximum loss given a confidence level $(1-\alpha)$ over a certain time horizon and can be expressed as the α-th percentile of the return distribution (Tsay, 2005). However, VaR at the institution level does not take into account the importance of externalities associated with systemic risk from other sectors or market participants. Furthermore, understanding VaR as the minimum estimated loss in a certain

[1] Through sovereign wealth funds, for example.
[2] Since its capital comes mainly from international reserves.
[3] For further detail about its investments the reader is encouraged to read these entities financial statements with its notes.

number of extreme events[4] it is valuable to understand its behavior, and how it relates to various systemic risk factors.

This article proposes and evaluates an econometric model that aims to clarify the relationship between the VaR of a portfolio and various factors that may affect it. This model, called VaR-UQR (VaR - Unconditional Quantile Regression), estimates the relationship between VaR and various risk factors, determining the effect of a change in the return of a factor, such as changes in the risk-free interest rate or movements from the financial system, on VaR. In this way, it is intended to answer the question of how exposed an asset, or in this case a portfolio, is to systemic risk events (a summary of models used in the analysis of financial risks may be found in Bisias et. al. (2012)) and determine if it is possible to make asset allocation decisions based on the exposure of a portfolio to such events. The econometric model proposed in this paper offers an extension to the unconditional quantile regression (UQR) estimation technique proposed by Firpo et. al (2009), which has been widely used in the literature on labor economics (several examples can be found in Casado and Simón (2015), Fortin, Lemieux, and Firpo (2011), and Lasso and Rodríguez (2018). This extension provides a perspective on the methodology in which its estimation relaxes the assumption of data independence[5], therefore, the paper contributes to the existing literature on models for estimating the risk of systemic events and on the econometric method itself. For its empirical validation, data of international reserves portfolios were used.

Some models that attempt to estimate the determinants of risk metrics for systemic events are the CoVaR, Co-Risk, Systemic Expected Shortfall and VAR for VaR[6]. However, the development of this new model is relevant given that the estimation of VaR and its determinants through quantile regressions, or other methods, requires algorithms and estimates that may be computationally costly, as explained by White et. al. (2015). Also, the estimates may be meaningless when estimating the effect of a change in a set of explanatory variables on the conditional percentile at a certain value of the explanatory variables and not on the unconditional percentile (Firpo et. al. 2009).

This paper is structured as follows. Section 2 introduces the concepts of Value at Risk, the Conditional Autorregressive Value at Risk (CAViaR) and Unconditional Quantile Regression (UQR). Section 3 describes the method for estimating VaR, CAViaR and proposes the VaR-UQR model. In section 4 a Monte Carlo experiment is performed to test the robustness of the VaR-UQR model. Section 5 presents an empirical application of the VaR-UQR model by applying this methodology to the estimation of the effect of movements from the financial system on the Value at Risk of international reserves portfolios. Section 6 presents the conclusions.

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[4] Usually the worst 5%, or 1% of total events, on a one-day horizon.

[5] This assumption specifies that the data used in the unconditional quantile regression model must be independent and identically distributed.

[6] Published in Adrian and Brunnermeier (2009), International Monetary Fund (2009), Acharya et. al. (2010) and White et. al. (2015), respectively.

2. Introduction to the concepts: Value at Risk, CAViaR, and unconditional quantile regression

i. Value at Risk

The most common risk measure for finding optimal portfolios is volatility, however, it is a stylized fact that the distribution of returns does not always behave symmetrically and may have heavy tails (Boothe and Glassman, 1987) (Jansen and De Vries, 1991), so this risk measure may be negligible (Campbell et al. 2000) and other measures, such as VaR, are very common in practice.

Several authors have proposed methods for finding efficient portfolios using VaR as a risk measure[7]. Specifically, Puelz (2001), Larsen et al. (2002) and Gaivoronski and Pflug (2005) propose algorithms to minimize VaR given a fixed level of profitability finding the efficient VaR-mean frontier. However, some optimization algorithms impose additional assumptions about the relationship between VaR and other risk measures to obtain the VaR-mean efficient frontier (e.g., Larsen et al. (2002)). Other algorithms can be computationally costly (Gaivoronski and Pflug, 2005).

In order to measure the contribution of buying an additional security in a portfolio to its VaR, Dowd (1999) proposes the measurement of incremental VaR (IVaR), which is defined as the difference between the VaR of a portfolio before and after the purchase of that security. In his book, Dowd (1999) shows that the IVaR is a function of the VaR of the new security multiplied by the Beta of the asset with respect to the portfolio, in the case that the returns are Gaussian. Tasche and Tibilleti (2003) extend this representation of the IVaR for returns with elliptical distributions. They also find two representations: one when a new asset is added to the portfolio and another when the weighting of the rest of the portfolio's assets is reduced to buy the new asset. The latter find that in the second case the IVaR is a function of the VaR of the new asset multiplied by the same Beta minus one. Wang (2002) explains how to calculate the IVaR through historical and Monte Carlo simulations, deriving the distribution of the IVaR for the second case. Although these methodologies estimate the Value at Risk, they do not consider external factors of a systemic nature that could affect the estimate of VaR.

ii. The CAViaR process

It is possible to understand CAViaR as the α-th percentile of the returns of a financial asset at the time t, contiditional on the information avaliable at $t-1$. The CAViaR model takes advantage of the fact that returns present volatility clusters, so that their distribution may be autocorrelated, which translates into autocorrelation in the tails (Engle and Manganelli, 2004). Thanks to this, it is a reliable method to estimate VaR[8]. Additionally, in order to estimate the effect of past events on VaR, Engle and Manganelli (2004) estimate the news impact curve (Engle and Ng, 1993), showing the effect of variations in the portfolio's lagged return on VaR. On the other hand, Chernozhukov and Umantsev (2001) estimate what they call conditional VaR through quantile regressions. In their model the dependent variable, the VaR, is explained

[7] See Campbell et al. (2000), Puelz (2001), Larsen et al. (2002) and Gaivoronski and Pflug (2005).
[8] See, for example, Engle and Manganelli (2004) and Mariño and Melo (2016).

by external and internal variables, such as lagged returns, returns of other assets and indexes. This means that the CAViaR is a specific case of Chernozhukov and Umantsev's (2001) model.

White et al. (2015) proposed the VAR method for VaR, which aims to analyze the dependence between the distribution tails of different assets. This method can be understood as an extension for several assets of the CAViaR model presented by Engle and Manganelli (2004). This notation allows the CAViaR of an asset to depend on the CAViaR of other assets, on their lag, and on the other information available up to the previous period. In addition, the authors derive a pseudo impulse-response function under a model configuration in which innovations are independent and identically distributed (iid) as standard normal.

The CAViaR model, and therefore the VAR for VaR, is estimated through quantile regressions, as explained in Koenker and Bassett (1978). However, the quantile regression model allows to do the inference of the conditional quantile of the dependent variable. The definition of the conditional quantile differs widely from the unconditional quantile, so the inference made may be misleading, as will be explained below.

iii. Unconditional quantile regression

Unconditional quantile regression is a special case of RIF (recentered influence function) regression. The RIF regression model was first proposed by Firpo et al. (2009) as a method to quantify the impact of change in the location of an explanatory variable on the unconditional, or marginal, distribution of an explanatory variable. Specifically, when the model is used to assess the effect of a change in location of an explanatory variable on the unconditional quantile of an explained variable, it is called unconditional quantile regression (UQR).

The UQR model differs from the traditional conditional quantile regression (QR) model proposed by Koenker and Bassett (1978) in that the estimators found through conditional quantile regression must be interpreted as the effect of a change in the explanatory variable on the quantile of the explained variable, conditional on certain value of the explanatory variable. In other words, the conditional quantile is the position of the explained variable relative to the rest of the observations that make up a similar set of values for the explanatory variables (Murakami and Seya, 2019). The estimators found through UQR can be interpreted as the effect of a change in the explanatory variable on the unconditional quantile of the explained variable. Firpo et al. (2009) call the estimators found through QR as "conditional quantile partial effect" (CQPE)[9].

The RIF regression model is based on the recentered influence function. The concept of influence function (IF) was developed by Hampel (1974), and explains the change in a function of a cdf (cumulative distribution function)[10] given a change in the distribution in the direction of another distribution, i.e. a directional derivative. Adding the function of the initial cdf gives the recentered influence function (RIF).

Firpo et al. (2009) show that UQPE can be easily estimated by performing OLS regression of the RIF variable, corresponding to the case of the quantile, on a set of exogenous explanatory

[9] The authors show that there is a direct relationship between the two methods, so that the "unconditional partial quantile effect" (UQPE), found by the UQR model, can be expressed as a weighted average of the CQPE on the distribution of the explanatory variables.
[10] Examples of a function of a cdf can be the mean, median, and percentiles, among others.

variables. Also, they develop the inference for UQPE when the assumption of independent and identically distributed variables holds.

Murakami and Seya (2019) develop the first extension of the UQR model applied to spatially correlated data. In order to avoid that the model error term has spatial correlation, they use an explanatory variable that captures this type of dependence.

3. Estimation methodologies: Value at Risk, CAViaR and VaR-UQR

Having introduced the relevant concepts for this document, this section describes the estimation procedure for each of them and mentions some of the differences between the methodologies.

i. Value at Risk

There are several methods for estimating VaR, one of them is parametric VaR, which assumes that the portfolio follows a known distribution, which in most cases, is assumed to be normal. Therefore, the problem of estimating parametric VaR is summarized in the calculation of the mean and standard deviation of portfolio returns, which is achieved by decomposing the portfolio return into different factors[11]. Since the publication of the RiskMetrics methodology by J.P. Morgan (Longerstaey & Spencer, 1996), the calculation of parametric VaR has been a market standard. However, it is a stylized fact that asset returns can have heavy tails and asymmetry, making Monte Carlo VaR calculation method more robust (Dowd 1999)[12].

To calculate the VaR by Monte Carlo simulation (MCS), a return path is simulated for each of the $i = 1, \dots, n$ securities in the portfolio. Then, using the simulated returns and weights of each asset at the initial moment, it is possible to calculate the return of the portfolio at the final moment. This process must be repeated p times. With p portfolio returns simulations it is possible to calculate the portfolio VaR as the percentile α of the simulation set. Since the securities in the portfolio are correlated, the return paths must be simulated together. This is achieved using the following formula:

$$\begin{bmatrix} s_{1,w} \\ s_{2,w} \end{bmatrix} = \begin{bmatrix} \overline{r_{1,w}} \\ \overline{r_{2,w}} \end{bmatrix} + A \begin{bmatrix} Z_{1,w} \\ Z_{2,w} \end{bmatrix} \quad (1)$$

Where $s_{i,t}$ is the simulated return of the asset i over the horizon w, $\overline{r_{\iota,w}}$ is the fixed return of the asset i[13] over the horizon w[14], $Z_{i,t}$ is a realization of a standard normal random variable and A is the lower triangular matrix of the cholesky decomposition of the covariance variance matrix of the assets returns.

[11] In the case of exposure to derivatives and other instruments on which the portfolio risk factors are not linear, it is necessary to use approaches that may also fall under the assumption of normality. An example of this is the Delta-Normal approach for options.
[12] Although, according to the Central Limit Theorem the sum of random variables is distributed normally, if the assets that make up the portfolio are not sufficiently independent, the assumption of normality about the portfolio's returns could be erroneous (Dowd, 1999).
[13] It can be explained by coupons or dividends, among others.
[14] For simplicity, daily returns will be used.

This method and other conventional methods omit the fact that VaR may be affected by external factors, so below methods that do take these factors into account are presented.

ii. CAViaR

Chernozhukov and Umantsev (2001), in their paper, combine the concepts of Value at Risk and quantile regression, defined as in Koenker and Bassett (1978), to obtain a representation of Value at Risk, conditional on a set of information, in a recursive manner. The representation they use is the following:

$$v_t(\alpha) = \dot{\boldsymbol{X}}_t' \boldsymbol{a}(\alpha) + \boldsymbol{b}(\alpha) g_1(\boldsymbol{X}_t, \alpha) \tag{2}$$

$$g_1(\boldsymbol{X}_t, \alpha) = \boldsymbol{c}(\alpha) g_t(\boldsymbol{X}_{t-1}, \alpha) + g_2(\dot{\boldsymbol{X}}_t, d(\alpha)) \tag{3}$$

Where $v_t(\alpha) \equiv F_Y^{-1}(\alpha|\boldsymbol{X}_t)$ and F_Y the cdf of the random variable Y. $\dot{\boldsymbol{X}}_t$ is a set of variables available at time t, and/or its transformations $\boldsymbol{X}_{t-1}$ is the set of variables $\{\dot{\boldsymbol{X}}_k\}_{k=0}^{t-1}$ so that $\boldsymbol{X}_t$=$(\dot{\boldsymbol{X}}_t, \boldsymbol{X}_{t-1})$.

The CAViaR model can be seen as a special case of recursive conditional VaR and can be represented as follows, taking into account factors external and internal to the portfolio:

$$v_t(\boldsymbol{\beta}(\alpha)) = \beta_0(\alpha) + \sum_{l=1}^{L} \beta_l(\alpha) v_{t-l}(\boldsymbol{\beta}(\alpha)) + \sum_{m=1}^{M} \boldsymbol{\varphi}_m(\alpha) g(\boldsymbol{X}_{t-m}) \tag{4}$$

The coefficients can be found through quantile regressions, defined as in Koenker and Bassett (1978), solving the following optimization problem:

$$\min_{\boldsymbol{\beta}(\alpha)} \frac{1}{T} \sum_{t=1}^{T} [\alpha - I(y_t < v_t(\boldsymbol{\beta}(\alpha)))][y_t - v_t(\boldsymbol{\beta}(\alpha))] \tag{5}$$

Where T is the sample size, $\boldsymbol{\beta}(\alpha) = (\{\beta_l(\alpha)\}_{l=0}^{L}, \{\boldsymbol{\varphi}_m(\alpha)\}_{m=1}^{M})$, $v_t(\boldsymbol{\beta}(\alpha))$ refers to the conditional quantile, and $I(\cdot)$ is the indicator function.

This optimization problem can be solved through numerical methods such as Nelder-Mead simplex algorithm. This method, combined with the optimization problem posed, may present some drawbacks in practice such as the dependence on the initial values and the computational cost that will depend on the number of parameters to be estimated and the length of the data.

Engle y Manganelli (2004) show that it is possible to perform the inference of the CAViaR model through the distribution of $\widehat{\boldsymbol{\beta}}(\alpha)$, this is $\sqrt{T}\boldsymbol{A}_T^{-\frac{1}{2}}\boldsymbol{D}_T(\widehat{\boldsymbol{\beta}}(\alpha) - \boldsymbol{\beta}(\alpha))$ converges in distribution to $N(0, \boldsymbol{I})$ with:

$$\boldsymbol{A}_T \equiv E\left[T^{-1}\alpha(\alpha - 1) \sum_{t=1}^{T} \nabla' v_t\big(\boldsymbol{\beta}(\alpha)\big) \nabla v_t\big(\boldsymbol{\beta}(\alpha)\big)\right] \tag{6}$$

$$\boldsymbol{D}_T \equiv E\left[T^{-1} \sum_{t=1}^{T} h_t(0|\Omega_t) \nabla' v_t\big(\boldsymbol{\beta}(\alpha)\big) \nabla v_t\big(\boldsymbol{\beta}(\alpha)\big)\right] \tag{7}$$

Where $\nabla v(\boldsymbol{\beta}(\alpha))$ is the gradient vector of $v(\boldsymbol{\beta}(\alpha))$, $h_t(0|\Omega_t)$ the density of $\varepsilon_{t\alpha} = y_t - v_t(\boldsymbol{\beta}(\alpha))$ evaluated at zero and Ω_t the set of available information.

The sample estimator of $\boldsymbol{A}_T$ and $\boldsymbol{D}_T$ can be expressed as:

$$\widehat{\boldsymbol{A}}_T = T^{-1}\alpha(\alpha-1)\nabla' v\left(\widehat{\boldsymbol{\beta}}(\alpha)\right)\nabla v\left(\widehat{\boldsymbol{\beta}}(\alpha)\right) \tag{8}$$

$$\widehat{\boldsymbol{D}}_T = (2T\hat{c}_T)^{-1}\sum_{t=1}^{T} I(|y_t - v_t\left(\widehat{\boldsymbol{\beta}}(\alpha)\right)| < \hat{c}_T)\nabla' v_t(\boldsymbol{\beta}(\alpha))\nabla v_t(\boldsymbol{\beta}(\alpha)) \tag{9}$$

Where $\hat{c}_T$ is the bandwidth of the density of $\varepsilon_{t\alpha}$ that can be estimated, following Koenker R. (2005, pp. 81) and Machado and Silva (2013), as:

$$\hat{c}_T = \hat{\kappa}[\Phi^{-1}(\alpha + \hat{\varrho}_T) - \Phi^{-1}(\alpha - \hat{\varrho}_T)] \tag{10}$$

Where $\hat{\kappa}$ is defined as the median of the absolute deviation of the residuals of the quantile regression of α-th percentile and with

$$\hat{\varrho}_T = T^{-\frac{1}{3}}\left(\Phi^{-1}\left(1-\frac{0.05}{2}\right)\right)^{\frac{2}{3}}\left(\frac{1.5(\phi(\Phi^{-1}(\alpha)))^2}{2(\Phi^{-1}(\alpha))^2+1}\right)^{\frac{1}{3}} \tag{11}$$

Where $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability density function and the cumulative probability function of a standard normal, respectively.

iii. VaR-UQR

Although this method does not directly calculate VaR, it estimates the relationship between VaR and external and internal factors to the portfolio based on the unconditional quantile regression proposed by Firpo et. al (2009). This method consists of running a regression of the recentered influence function on a set of variables. The recentered influence function can be expressed as:

$$RIF(Y;q_\alpha) = q_\alpha + \frac{\alpha - I(Y \le q_\alpha)}{f_Y(q_\alpha)} \tag{12}$$

Where $q_\alpha \equiv F_Y^{-1}(\alpha)$ and $f_Y(\cdot)$ is the pdf of the random variable Y.

The UQPE can be estimated using the RIF-OLS method, which consists of estimating the coefficients of the random variable $RIF(Y;q_\alpha)$ on a set of variables $\boldsymbol{X}$. It is also possible to estimate the UQPE through logistic regression of $I(Y > q_\alpha)$ on $\boldsymbol{X}$, by first estimating the average marginal effects, and then multiplying them by $\frac{1}{f_Y(q_\alpha)}$.

Estimating the RIF-OLS model is equivalent to estimating a linear probability model since the dependent variable is a binary response; however, this model is known to be consistent and more stable than the logistic regression model (Murakami and Seya, 2019). Additionally, the estimation of coefficients by OLS does not need initial values to solve the optimization problem and its solution is unique, in contrast to the CAViaR optimization problem.

Recently, Dong, Li, and Yoon (2019) and Huang (2018) applied the RIF regression method to assets returns data. However, they omitted the assumption that the data, in the model proposed by Firpo et. al. (2009), should be independent and identically distributed.

Following Murakami and Seya (2019), in order to adapt the model to this type of data, the lag of the asset returns is added as explanatory variable[15]. What is economically implied by adding this variable is that the distribution tail depends on the distribution in the previous period, reflecting possible volatility clusters.

Firpo et al. (2009) develop the inference in their applications through bootstrapping. Since the sample may present autocorrelation, the boostrapping method known as block resampling will be applied, and, specifically, stationary resampling will be applied to the observations (Politis and Romano, 1994)[16]. Under this resampling method, the original sample is divided into j overlapping blocks of length b. The length of the blocks is distributed geometrically with average b^0. The blocks are then randomly taken with replacement and pasted at the end of the new sample until it has a length T. This process is repeated r times. Politis and White (2004) propose as optimal estimator of b^0:

$$\hat{b} = \left(\frac{2\hat{G}^2}{\hat{D}}\right)^{1/3} T^{1/3} \tag{13}$$

With $\hat{G} = \sum_{n=-L}^{L} \lambda\left(\frac{n}{L}\right)|n|\hat{R}(n)$ and $\hat{D} = (4\hat{g}^2(0) + \frac{2}{\pi}\int_{-\pi}^{\pi}(1+\cos(\omega))\hat{g}^2(\omega)d\omega$.

$L = 2\hat{m}$, where $\hat{m}$ is the lag after which the autocovariance ceases to be significant or is negligible[17], $\hat{R}(\cdot)$ is the sample autocovariance function, $\hat{g}(\omega) = \sum_{n=-L}^{L} \lambda\left(\frac{n}{L}\right)\hat{R}(n)\cos(\omega n)$ and:

$$\lambda(\eta) = \begin{cases} 1 & if\ |\eta| \in [0,1/2] \\ 2(1-|\eta|) & if\ |\eta| \in [1/2,1] \\ 0 & otherwise \end{cases} \tag{14}$$

This methodology is the one proposed in this document for the estimation of the marginal effects of change in risk factors on Value at Risk. To test the robustness of the proposed method, two Monte Carlo experiments are conducted and presented in the following section.

4. Montecarlo experiment

In order to test the performance of the VaR-UQR model against other models and its validity, this section proposes several specifications for the data generating process (DGP), where the dependent variable presents autocorrelation on the tails of its distribution. Then, CAViaR and VaR-UQR models are compared through Monte Carlo experiments. In these DGPs, the dependent variable presents first-order autocorrelation with the lag of the latent variable. To evaluate the models, the bias of the estimators of each model, their standard deviation and the root of the root mean square error (RMSE) are measured.

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[15] If necessary, it is possible to use more lags, however, this will depend on the temporary persistence of the data. A statistical test on the UQR model residuals would be the best way to determine the number of lags needed, however, their development and use are beyond the scope of this document.
[16] The stationary resampling method has a mean square error (at the limit when T tends to infinity) greater than the common block resampling when using an optimal block length. However, the optimal block size is not known in practice and it is preferable to use the stationary resampling method, as it is less sensitive to specification errors in block size (Politis and Romano, 1994).
[17] Politis and White (2004) propose to identify $\hat{m}$ from analyzing the correlogram of the variable in question.

i. ARX(1)

First, an autoregressive DGP of order 1 with an exogenous variable is proposed. In this case, both X and Y present autocorrelation. X is generated by an AR(1) process with a disturbance distributed $N(0, 0.02^2)$. The intercept is 0.02 and the autoregressive coefficient is 0.5. $y_t = 0.05 + 0.5y_{t-1} + 0.3x_t + \varepsilon_t$, with $\varepsilon_t \sim N(0, 0.05^2)$. The number of replications is 5,000 and the sample sizes used are 750 y 1,250.

In this process the sample is generated in all its percentiles by the same DGP and does not present heteroscedasticity. Therefore, the estimators of the coefficients in all percentiles of the CAViaR and VaR-UQR regressions are the same. Furthermore, in a linear model, as described above, UQPE is equivalent to CQPE when the model perturbation is independent of the regressors (Firpo et. al, 2009).

The estimated CAViaR is $v_t(\alpha) = \beta_0(\alpha) + \beta_1(\alpha)v_{t-1}(\alpha) + \varphi_1(\alpha)y_{t-1} + \varphi_2(\alpha)x_t$. The estimated RIF is $RIF(y_t; q_\alpha) = \gamma_0(\alpha) + \gamma_1(\alpha)y_{t-1} + \gamma_2(\alpha)x_t$.

Tables 1 and 2 of the appendix contain the results for Monte Carlo with a sample size of 750. Tables 3 and 4 contain the results with a sample size of 1,250.

From the results, it is possible to notice that the estimators of the models tend to present a greater bias in the tails, as well as greater standard deviation and greater RMSE. This is consistent with the findings of Manganelli and Engle (2001) who explain that this behavior is due to the fact that few observations are available at these probability levels. Looking at the results obtained by Dong, Li, and Yoon (2019) it is possible to think that the U and U-inverted shape of the estimators across the quantile found by these authors may be due to the bias in the distribution tails when applying the models. As the sample size increases from 750 to 1,250, this effect fades away, since the models have more observations available and the number of parameters to be estimated are kept constant.

For this DGP the CAViaR model estimates in a better way the parameters (it has less bias, standard deviation and RMSE), however, the VaR-UQR model tends to be robust since when increasing the sample size the estimators by this method improve. Besides, it has the advantage of being easily estimable through OLS.

It should be emphasized that this DGP does not present a differentiated behavior through its percentiles. For this reason, it is pertinent to propose a model that takes into account this possible behavior, which is the main reason for using quantile regression techniques. Next, we propose a DGP in which only one percentile of the distribution of the dependent variable is affected by the independent variables through the proposed coefficients.

ii. Autoregressive RIF

In this simulation, the variable Y is simulated by inverting the $RIF(Y; q_\alpha)$ function. Since $RIF(Y; q_\alpha)$ is not continuous, Y must be generated from a simulation based on the values obtained when simulating $RIF(Y; q_\alpha)$.

Let's assume that X is generated by an AR(1) process with a known distribution disturbance and an autoregressive coefficient of 0.5. The variable $RIF(y_t; q_\alpha)$ is calculated in such a way that:

$$RIF(y_t; q_\alpha) = \begin{cases} q_\alpha + (\alpha - 1)/f_Y(q_\alpha) & if\ \gamma_1(\alpha)y_{t-1} + \gamma_2(\alpha)x_t + \varepsilon_t \leq q_\alpha \\ q_\alpha + \alpha/f_Y(q_\alpha) & otherwise \end{cases} \quad (15)$$

In the same way y_t is simulated in such a way that:

$$y_t = \begin{cases} F_Y^{-1}(Z) & if\ \gamma_1(\alpha)y_{t-1} + \gamma_2(\alpha)x_t + \varepsilon_t \leq q_\alpha \\ F_Y^{-1}(Z') & otherwise \end{cases} \quad (16)$$

With $Z \sim unif(0, \alpha)$ and $Z' \sim unif(\alpha, 1)$.

This DGP specification implies that equality is maintained in the expression $RIF(y_t; q_\alpha) = q_\alpha + \frac{\alpha - I(y_t \leq q_\alpha)}{f_Y(q_\alpha)}$. In addition, this DGP does not impose any relationship between the explanatory variables and the conditional percentile, so it would be expected that only the unconditional percentile reflects the relationship proposed through $\gamma_i(\alpha)$ with the explanatory variables. What this implies is that the results of the CAViaR and VaR-UQR models may differ.

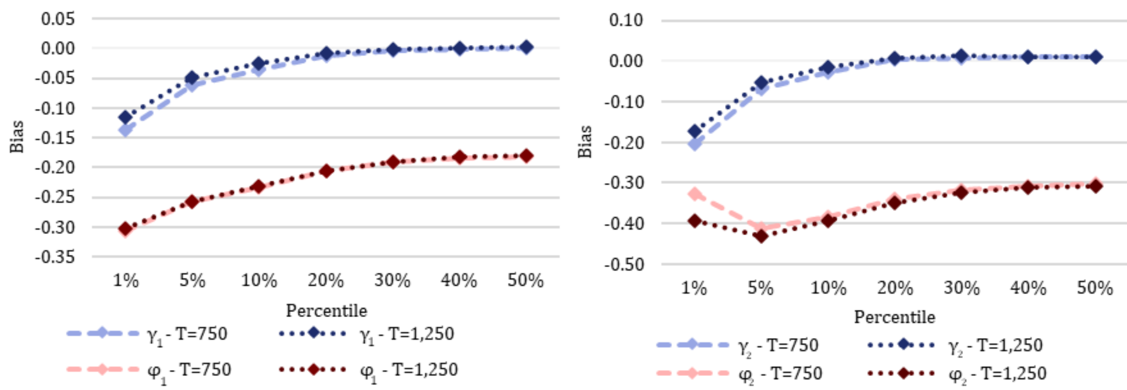
The estimated models are: $v_t(\alpha) = \beta_0(\alpha) + \beta_1(\alpha)v_{t-1}(\alpha) + \varphi_1(\alpha)y_{t-1} + \varphi_2(\alpha)x_t$ and $RIF(y_t; q_\alpha) = \gamma_0(\alpha) + \gamma_1(\alpha)y_{t-1} + \gamma_2(\alpha)x_t$. The number of replications is 10,000 for samples of 750 and 1,250 observations. The disturbance of X is distributed $N(0, 0.01^2)$, $\varepsilon_t \sim N(0, 0.05^2)$, $\gamma_1(\alpha) = 0.5$ and $\gamma_2(\alpha) = 0.01/\sigma_X$.

Tables 5 and 6 of the appendix present the results for Monte Carlo with a sample size of 750. Tables 7 and 8 contain the results with a sample size of 1,250. Figure 1 shows the bias of the estimators when applying the CAViaR and VaR-UQR models to the autoregressive DGP RIF for percentiles 1% to 50%. The blue line shows the bias of the estimators of the VaR-UQR model with sample sizes 750 and 1,250. In red, the bias of the CAViaR model estimators is presented with sample sizes 750 and 1,250. The left graph shows the bias of the estimators corresponding to the lagged return and the right graph shows the bias of the estimators corresponding to the exogenous variable X.

In this experiment, the VaR-UQR model does a good job estimating the coefficients since the bias, standard deviation, and RMSE fade as the sample size increases. Again, on the tails the metrics are not as good as in the center of the sample given the small amount of data. Note that the bias of the estimators is virtually null near the median, and the decrease in bias as sample size increases is significant for percentiles near the tail.

The CAViaR model, on the other hand, produces different estimates which can lead to misleading inference. Although accuracy seems to increase by using percentiles closer to the median, metrics do not improve when increasing sample size. This is because the CAViaR model attempts to estimate CQPE. For this reason, this methodology is not appropriate for estimating the simulated effect, while the VaR-UQR methodology is.

Figure 1 – Estimates bias



Source: author's calculations.

In summary, the VaR-UQR model offers a robust estimation of UQPE as it reduces the bias, volatility and root mean square error metrics when the data generating process meets the specifications mentioned above. It is also easily estimated by ordinary least squares (OLS), which is an advantage over the estimation method of the CAViaR model. On the other hand, the CAViaR model allows, only in some cases, to reduce the evaluated metrics and, therefore, only in those cases it would lead to valid conclusions on the inference. In addition, to estimate the CAViaR model it is necessary to have precise initial values for the optimization problem, which in practice are not known.

Once the methodology proposed in this paper for the estimation of the marginal effects of different risk factors on the Value at Risk has been explained and the robustness of the methodology compared to the CAViaR model has been proven, an empirical exercise is carried out in which the VaR-UQR model is used on international reserves portfolio data.

5. Application of the VaR-UQR methodology to international reserve portfolio data

This section uses the VaR-UQR and CAViaR models on international reserves portfolio data, mostly used by central banks, sovereign and supranational funds, to estimate the effect of increases and decreases in the returns of certain assets and risk factors on Value at Risk.

As explained above, return distribution tails can be autocorrelated (Engle and Manganelli, 2004). The CAViaR model isolates this correlation; however, the distribution tail may be correlated with other variables[18]. White, Kim, and Mangenelli (2015) address this problem with the VAR for VaR methodology, leaving the CAViaR of certain variables dependent on each other, however, this implies to jointly estimate several quantile regression parameters, which is computationally costly. Adrian and Brunnermeier (2009), International Monetary Fund (2009) and Acharya et. al. (2010) do similar work for financial institutions with the CoVaR, Co-Risk and Systemic Expected Shortfall models, respectively. These models are applied in the context of the propagation of negative shocks among financial institutions, i.e., the propagation of tail events.

[18] When including explanatory variables in the CAViaR regression, the estimated CAViaR may be erroneous as it represents the conditional percentile on the value of the explanatory variables. For this reason, an interpretation of the coefficients estimated by means of CAViaR is not provided.

The following application of the VaR-UQR model can be built around this methodological framework, as it explains how a systemic financial institution can be affected by external market shocks, and particularly, aims to answer the question of how tail events spread.

i. The data

International reserve portfolios are characterized by their conservative exposure to financial risk. For this reason, they are usually fixed-income portfolios, concentrated in securities with no credit risk (such as U.S. government bonds), with a low volatile yield, low duration and, commonly, are counter-cyclical. Similarly, they may be exposed to securities with low credit risk, such as mortgage backed securities (MBS) issued by government agencies, or securities issued by issuers with a credit rating within the investment grade spectrum.

Data were taken from a typical portfolio of international reserves (portfolio 1) for the period June 30, 2017 to December 31, 2019. This portfolio has exposure to U.S. treasuries (with an average 41% market share[19]) and Treasury Inflation Protected Securities (TIPS) (with an average 9% market share), bonds (average 10% market share) and money market[20] instruments (average 41% market share) issued by investment grade corporations and quasi-governments (average 8% market share)[21]. The above instruments are denominated in U.S. dollars.

Considering that this portfolio has greater exposure to money market instruments than to bonds, by breaking down securities that are not backed, implicitly or explicitly, by the U.S. government, the short-term ratings (A-1 and A-2) have an average share of 79%. The rest is 8% average participation in AA ratings and 14% in A ratings. Their average duration is 1.1 years.

The sample contains 645 observations of daily returns, in basis points. The Box-Jenkins analysis, Augmented Dickey-Fuller and Ljung-Box tests suggest that the returns are autocorrelated in order 1. The estimated parameter for the AR(1) model is -0.124, significant at a 5% probability level.

Additionally, data were taken from another typical portfolio of international reserves (portfolio 2) for the period from March 31, 2015 to December 31, 2019. This portfolio has exposure to U.S. treasuries (with an average share of 28%) and TIPS (with an average share of 6%), MBS (7%) and U.S. agency bonds (2%), bonds (24%) and money market instruments (26%) issued by investment grade corporations and quasi-governments (10%), denominated in U.S. dollars.

In terms of composition by credit rating, when breaking down securities that are not backed, implicitly or explicitly, by the U.S. government, short-term ratings (A-1 and A-2) have an average share of 48%. The remaining 6% average participation in AAA ratings and 14% in AA ratings and 33% in A ratings. The average duration of this portfolio is 0.86 years.

[19] Average of semi-annual market values.

[20] Money market instruments are mostly securities that are bought at a discount, or certificates of deposit.

[21] These values do not necessarily add up to 100% because negative cash, i.e. accounts payable, may exist between the transaction date and the settlement date of the purchase transactions.

The sample contains 1,225 observations of daily returns, in basis points. Box-Jenkins analysis, Augmented Dickey-Fuller and Ljung-Box tests suggest that the returns in this portfolio are not generated by an autoregressive process.

To estimate the CAViaR and VaR-UQR models, two sets of variables were used as explanatory variables. The first set, naturally, is the indexes from which the reference portfolio is composed[22]. It would be expected that portfolio risk is explained by one, or some, of these indexes since they are the basis for the selection of the securities that make up each portfolio. The second set of variables is the risk factors returns of[23] fixed income securities denominated in USD. These factors represent the return, and therefore the risk, specific to each asset type. Correctly estimating factor returns allows the performance of securities in each economic sector to be quantified.

To avoid seasonal effects generated by interest accrual, the return of shorter duration indexes, such as the 0-1 year Treasury Index and the 3-month LIBOR Index, is taken as the return of the index price.

Risk factor returns can be calculated through factor models that attempt to decompose the return of a security into different factors with or without economic interpretation. Among the pioneer models in this area can be found the CAPM[24], which explains the return of a portfolio as a function of the risk-free rate and the excess return of a market index; also, the APT model (Ross, 1976) explains the return of a well-diversified portfolio in terms of the expected return of its securities plus a component explained by the return of a factor.

Fama and French (1992) assumed that there are observable factors that can explain the return on a stock, such as the size of a company, its leverage, among others. Therefore, when running a cross-sectional regression of the return of a set of stocks on these factors the returns associated with those factors are obtained as the estimated betas. This principle is maintained when trying to explain the return of fixed-income securities through factors. A bond has as observable factors its duration, exposure to changes in the slope and convexity of the yield curve[25] and margin duration associated with the exposure (1 for duration if exposed or 0 if not) to different sectors or credit ratings.

Changes in duration factor returns are generated by parallel movements in the yield curve, so if the yield curve goes up, you would expect to see negative returns on this factor and vice versa. The changes in the return of the yield curve slope factor are mostly explained by the changes in the rates of longer term bonds (more than 10 years at maturity), since the exposure to this factor is greater in these bonds. Finally, changes in the return of the convexity factor of the yield curve are mostly explained by changes in the rates of medium-term bonds (about 5 years at maturity) (Amenc and Le Sourd, 2005 pp. 242).

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[22] The source of the indexes is Bloomberg, except for the corporate index which can be found at https://fred.stlouisfed.org/series/BAMLC1A0C13YEY.

[23] These returns were provided by Wilshire Associates.

[24] The CAPM model was published in Sharpe (1964) Lintner (1965) and Mossin (1966).

[25] The yield curve is the US government's treasury curve.

In addition, the spreads on USD-denominated bonds are time-dependent, i.e. the longer the time to maturity of a bond, the higher the spread; therefore, this is an observable factor[26], similar to exposure to changes in the slope of the yield curve.

On the other hand, bonds with uncertain cash flows, such as MBS, may be exposed to additional factors such as prepayment elasticity, i.e. the change in price explained by the change prepayment speed. Bonds with options, such as call or put, are exposed to changes in the price of that option, which depends on the volatility of interest rates.

Running a cross-sectional regression with returns of a bonds from an index on exposure to these factors would result in the returns of the factors, represented in each estimated beta.

Table 9 in the appendix summarizes some descriptive statistics of the variables. It is possible to highlight that: 1) the returns of the agency sector have little volatility, however, their kurtosis is high compared to other sectors, indicating that the probability of extreme events is higher. 2) Among the agencies that issue MBS, Fannie Mae stands out with a high kurtosis. The same is true for prepaiment elasticity returns. 3) Among the indexes, the most volatile are the MBS and TIPS, although the indexes with the highest kurtosis are those with short durations - 0-1 year Treasuries and 3-month Libor.

ii. Estimation and results

For the inference of the VaR-UQR model the number of replications for bootstrapping is equal to 10,000. The α, or significance level, used is 5%.

For the CAViaR model the optimization is done using the Nelder-Mead simplex algorithm. To find the initial values of the optimizer, the loss function is evaluated on a set of 10^6 possible initial values generated from a uniform random variable with support between -2 and 2. Then, the optimizer is applied to the initial values of the best 20 values obtained from evaluating the loss function. The final initial values will be those that produce the best value of the optimizer. The relative tolerance of the optimizer is 10^{-100}. However, sometimes using the VaR-UQR model estimators as initial values for the optimizer produces better estimates than performing the process described above.

Figures 2 and 3 show the results of the CAViaR regression and the return of the portfolios with its historical VaR. The results of the VaR-UQR model are shown in tables 10, 12, 14 and 16 of the appendix and the results of the CAViaR model are shown[27] in tables 11, 13, 15 and 17 of the appendix. In the figures it is possible to see that volatility tends to cluster over time, so that once the VaR is exceeded, the probability of similar events occurring in the future is greater. The lagged portfolio return variable captures this effect, giving a negative and significant coefficient in all the models estimated. Understanding volatility as the average deviation from the mean, and considering the fact that when the VaR is exceeded the volatility is increasing and changing from one cluster to another, it is possible to explain this coefficient given that prior to an event when the VaR is exceeded (a tail event) the returns are increasingly far from their mean. This deviation from the average, in the case of these portfolios is upwards (the return increases), which is explained by episodes of risk aversion, in which investors seek

[26] Their return is represented by the factor called "Term Spread".

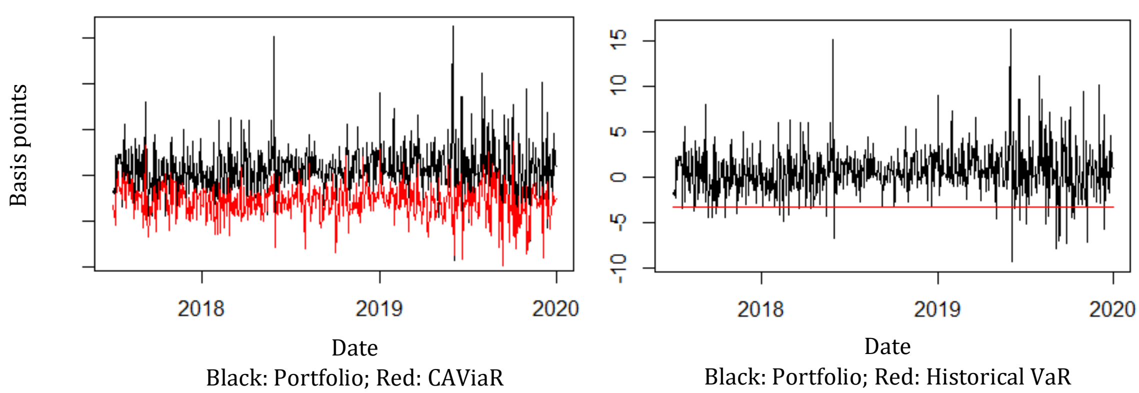
[27] In this section the coefficients found through the VaR-UQR model are interpreted. Those found through the CAViaR model are presented so that the reader can compare the estimates.

refuge in safe assets, such as US treasuries, which, due to greater demand, have a substantial increase in price, which makes these securities expensive. This increase in the return on safe assets is corrected, usually the next day, generating an event in which the VaR is exceeded.

The result of the regressions on the indexes is very much in line with what would be expected. It is remarkable how the TIPS and the corporates have a directly proportional effect on the portfolio's VaR, that is, when the return of these indexes falls, it generates that the portfolio's VaR is even deeper. On the one hand, TIPS, despite being US government bonds, have a very high volatility, so the exposure of both portfolios is sufficient for their risk to be affected by this type of securities. On the other hand, corporate bonds comprise many economic sectors, which can reflect systemic risk events. In this case, the fact that the sign of the estimated ratio is positive on corporate indicates that the portfolio risk, represented by the VaR, may be affected by any of these sectors that reflect systemic risk.

In order to analyze which specific sectors are affecting the VaR of the portfolios the regression was run on the risk factor returns. In principle, in both portfolios was found that their VaR is affected by changes in the risk-free rate, but in different ways. The VaR of portfolio 1 presents sensitivity for all possible changes in returns of risk-free rate factors (Duration, D2 and D3), while the VaR of portfolio 2 is only affected by changes in convexity of the yield curve. Therefore, the VaR varies proportionally to the changes in returns generated by changes in bond rates of around 5 years, keeping bonds with other maturities stable. What this implies is that portfolios are exposed to sudden increases in the risk-free rate. A similar argument can be built around the return of the prepaid elasticity of Ginnie Mae bonds. As interest rates rise in the United States, it is expected that MBS prepayments will decline and that cash flows from these prepayments will decline as well, so the price of these bonds will be lower. Finding a positive coefficient in the return of the portfolio 1 on this factor reaffirms that the portfolio's VaR is affected by increases in risk-free interest rates.

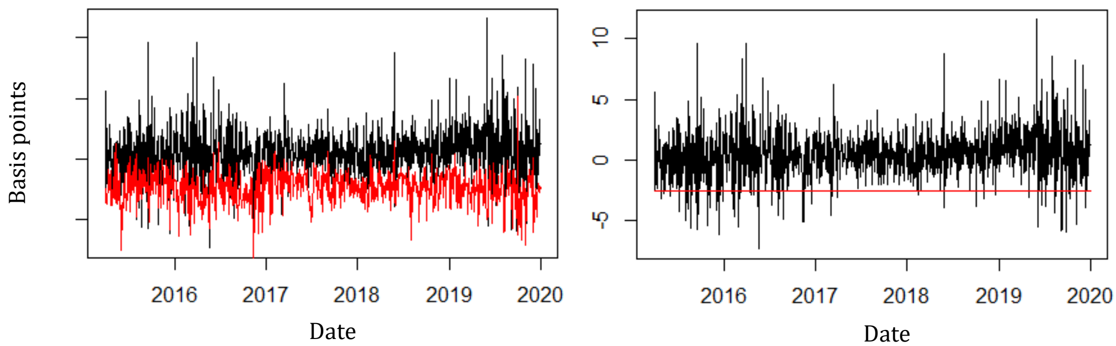
Figure 2 - Returns, estimated CAViaR and historical VaR of portfolio 1.



Source: author's calculations

Analyzing in detail the shocks coming from the financial and banking system, it is important to highlight that the VaR of portfolio 1 is not explained by these sectors; however, the VaR of portfolio 2 is, although in an inversely proportional way. This effect may be desirable since it implies that when the portfolio experiences tail events, usually the return of the banking sector increases, this indicates that the portfolio behaves counter-cyclically related to this sector in tail events. This phenomenon can be explained by exposure to systemically important banks[28], which, due to the high regulatory standards required for them, are safer than their peers.

Figure 3 - Returns, estimated CAViaR and historical VaR of portfolio 2.



Black: Portfolio; Red: CAViaR

Black: Portfolio; Red: Historical VaR

Source: author's calculations

Another very important sector for this type of portfolio is the supranational, since they also use international reserve portfolios and the exposure in their credit operations is, in most cases, directly with governments. In this case, the VaR of the two portfolios has a direct relationship with this sector. Since both portfolios have exposure to this sector and a similar composition of their assets, it is reasonable that their tail events are positively related to their returns.

Finally, exposure to other, more risky sectors may be affecting VaR, as is the case of portfolio 1 and the telephony and communications sector. On the other hand, the portfolio's VaR can behave in a counter-cyclical way against other sectors, as it is the case of portfolio 1 against Ginnie Mae's MBS and inflation and, portfolio 2 against Yankee bonds.

6. Conclusions

Through Monte Carlo experiments the RIF regression method, and particularly the unconditional quantile regression, was found to work for data with autocorrelation and reduced the bias, variance and mean square error when correctly specifying the model. Since this method can be estimated by OLS, it has the advantage of being computationally efficient rather than conventional methods, and it generates as a result Unconditional Partial Quantity Effects (UQPE), which reflect the effect of the change in one variable on the unconditional quantile of another.

Once the robustness of the VaR-UQR model, through the Monte Carlo experiment, was proved this methodology was applied to data of international reserves portfolio returns considering Value at Risk as the explained variable. Results are relevant in this empirical study because they may be used as a method to evaluate factors to which a portfolio or a financial asset is exposed. It was found that the Value at Risk of international reserves portfolios is exposed to changes in the risk-free interest rate; however, when analyzing changes arising from the financial and banking sector, the VaR of this type of portfolios is covered .

[28] The list of these banks is published at https://www.fsb.org/2019/11/2019-list-of-global-systemically-important-banks-g-sibs/.

RIF regression is still a topic on which there is a lot of work to be done and this article is a first step on all the possible applications that these types of regressions could have. It is relevant, in the future, to develop the distribution of the estimators found in order to make the inference more accurate. Furthermore, it is important for a future research work to explicitly consider the possible time-conditional heteroscedasticity[29] of the returns. Finally, another field in which RIF regression can be applied is on the determinants of financial asset volatility.

[29] As in ARCH and GARCH models, among others.

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Appendix

Results of Monte Carlo experiments

Table 1 - Monte Carlo ARX(1), sample size = 750. Autoregressive Coefficients.

α	φ_1					γ_1				
	Bias	Std.	RMSE	Perc. 5%	Perc. 95%	Bias	Std.	RMSE	Perc. 5%	Perc. 95%
1%	0.000	0.124	0.124	0.289	0.707	-0.065	0.186	0.197	0.200	0.770
5%	0.000	0.074	0.074	0.378	0.620	-0.030	0.102	0.107	0.322	0.658
10%	-0.004	0.062	0.062	0.395	0.599	-0.017	0.080	0.081	0.363	0.623
20%	-0.004	0.052	0.052	0.409	0.583	-0.002	0.063	0.063	0.397	0.604
30%	-0.003	0.049	0.049	0.415	0.577	0.004	0.055	0.056	0.416	0.596
40%	-0.003	0.047	0.047	0.419	0.573	0.008	0.052	0.053	0.424	0.597
50%	-0.003	0.046	0.046	0.423	0.572	0.010	0.051	0.051	0.428	0.594
60%	-0.003	0.047	0.047	0.420	0.575	0.010	0.052	0.053	0.425	0.599
70%	-0.002	0.048	0.048	0.419	0.579	0.007	0.055	0.056	0.418	0.600
80%	-0.002	0.052	0.052	0.414	0.584	0.002	0.062	0.062	0.406	0.608
90%	-0.002	0.062	0.062	0.394	0.598	-0.013	0.080	0.081	0.364	0.625
95%	-0.003	0.076	0.076	0.376	0.623	-0.025	0.103	0.106	0.323	0.661
99%	-0.002	0.131	0.131	0.285	0.714	-0.062	0.189	0.199	0.202	0.793

Source: author's calculations

Table 2 - Monte Carlo ARX(1), sample size = 750. Coefficients of x_t.

α	φ_2					γ_2				
	Bias	Std.	RMSE	Perc. 5%	Perc. 95%	Bias	Std.	RMSE	Perc. 5%	Perc. 95%
1%	0.018	0.257	0.257	-0.108	0.745	-0.034	0.334	0.335	-0.239	0.840
5%	0.011	0.158	0.158	0.049	0.562	-0.010	0.196	0.196	-0.014	0.623
10%	0.008	0.131	0.131	0.088	0.522	-0.005	0.160	0.160	0.036	0.566
20%	0.007	0.112	0.112	0.122	0.493	0.004	0.131	0.131	0.089	0.522
30%	0.007	0.102	0.103	0.133	0.477	0.009	0.119	0.120	0.117	0.508
40%	0.004	0.099	0.099	0.141	0.467	0.008	0.110	0.110	0.130	0.494
50%	0.003	0.100	0.100	0.140	0.468	0.009	0.110	0.110	0.130	0.489
60%	0.004	0.101	0.101	0.140	0.470	0.010	0.112	0.112	0.126	0.492
70%	0.003	0.103	0.103	0.134	0.470	0.010	0.119	0.119	0.116	0.504
80%	0.004	0.110	0.110	0.125	0.485	0.005	0.130	0.130	0.092	0.520
90%	0.001	0.131	0.131	0.086	0.521	-0.005	0.159	0.159	0.044	0.562
95%	0.001	0.164	0.164	0.031	0.579	-0.012	0.196	0.196	-0.028	0.622
99%	0.001	0.271	0.271	-0.139	0.763	-0.032	0.342	0.343	-0.233	0.854

Source: author's calculations

Table 3 - Monte Carlo ARX(1), sample size = 1,250. Autoregressive Coefficients.

α	φ_1					γ_1				
	Bias	Std.	RMSE	Perc. 5%	Perc. 95%	Bias	Std.	RMSE	Perc. 5%	Perc. 95%
1%	0.001	0.100	0.100	0.337	0.666	-0.053	0.151	0.160	0.246	0.720
5%	0.000	0.058	0.058	0.405	0.593	-0.022	0.080	0.083	0.357	0.618
10%	-0.001	0.048	0.048	0.419	0.577	-0.011	0.063	0.064	0.392	0.597
20%	-0.001	0.040	0.040	0.433	0.566	0.000	0.049	0.049	0.422	0.584
30%	-0.001	0.037	0.038	0.436	0.560	0.005	0.043	0.043	0.434	0.578
40%	-0.001	0.036	0.036	0.438	0.558	0.008	0.040	0.041	0.442	0.575
50%	-0.002	0.035	0.036	0.438	0.555	0.009	0.040	0.041	0.442	0.574
60%	-0.002	0.036	0.036	0.438	0.557	0.009	0.040	0.041	0.443	0.575
70%	-0.002	0.037	0.037	0.436	0.558	0.005	0.044	0.044	0.435	0.581
80%	-0.002	0.040	0.040	0.432	0.565	0.000	0.050	0.050	0.421	0.585
90%	-0.002	0.049	0.049	0.416	0.577	-0.012	0.063	0.064	0.390	0.599
95%	-0.003	0.059	0.059	0.401	0.593	-0.023	0.082	0.085	0.353	0.622
99%	-0.005	0.102	0.102	0.331	0.664	-0.053	0.152	0.161	0.246	0.721

Source: author's calculations

Table 4 - Monte Carlo ARX(1), sample size = 1,250. Coefficients of x_t.

α	φ_2					γ_2				
	Bias	Std.	RMSE	Perc. 5%	Perc. 95%	Bias	Std.	RMSE	Perc. 5%	Perc. 95%
1%	0.008	0.203	0.203	-0.033	0.638	-0.038	0.254	0.257	-0.136	0.701
5%	0.006	0.123	0.124	0.096	0.502	-0.010	0.150	0.150	0.050	0.540
10%	0.003	0.104	0.104	0.132	0.476	-0.005	0.123	0.123	0.092	0.501
20%	0.002	0.087	0.087	0.155	0.448	0.002	0.101	0.101	0.136	0.470
30%	0.002	0.080	0.080	0.170	0.434	0.004	0.093	0.093	0.152	0.458
40%	0.000	0.079	0.079	0.173	0.430	0.006	0.088	0.088	0.164	0.451
50%	0.001	0.080	0.080	0.173	0.434	0.006	0.085	0.086	0.169	0.447
60%	0.002	0.081	0.081	0.169	0.436	0.006	0.088	0.088	0.161	0.452
70%	0.002	0.082	0.082	0.168	0.437	0.006	0.092	0.092	0.153	0.459
80%	0.003	0.086	0.086	0.161	0.448	0.004	0.102	0.102	0.135	0.471
90%	0.002	0.102	0.102	0.132	0.475	-0.002	0.124	0.124	0.095	0.504
95%	0.001	0.127	0.127	0.092	0.513	-0.011	0.152	0.152	0.045	0.547
99%	0.006	0.219	0.219	-0.047	0.668	-0.029	0.262	0.263	-0.141	0.723

Source: author's calculations

Table 5 - Monte Carlo RIF autoregressive, sample size = 750. Autoregressive coefficients.

α	φ1					γ1				
	Bias	Std.	RMSE	Perc. 5%	Perc. 95%	Bias	Std.	RMSE	Perc. 5%	Perc. 95%
1%	-0.306	0.093	0.320	0.042	0.345	-0.138	0.204	0.246	0.066	0.719
5%	-0.258	0.054	0.263	0.155	0.332	-0.061	0.112	0.128	0.254	0.628
10%	-0.233	0.045	0.237	0.195	0.341	-0.036	0.082	0.089	0.330	0.598
20%	-0.206	0.038	0.209	0.232	0.358	-0.013	0.061	0.062	0.388	0.587
30%	-0.191	0.037	0.194	0.250	0.370	-0.004	0.052	0.052	0.410	0.582
40%	-0.183	0.036	0.187	0.258	0.377	0.000	0.047	0.047	0.423	0.579
50%	-0.181	0.036	0.185	0.261	0.378	0.001	0.046	0.046	0.426	0.577

Source: author's calculations

Table 6 - Monte Carlo RIF autoregressive, sample size = 750. Coefficients of x_t.

α	φ2					γ2				
	Bias	Std.	RMSE	Perc. 5%	Perc. 95%	Bias	Std.	RMSE	Perc. 5%	Perc. 95%
1%	-0.326	0.502	0.599	-0.286	1.317	-0.205	0.699	0.729	-0.360	1.893
5%	-0.411	0.301	0.510	-0.026	0.957	-0.067	0.409	0.415	0.149	1.493
10%	-0.384	0.246	0.456	0.088	0.898	-0.027	0.325	0.326	0.320	1.373
20%	-0.341	0.212	0.401	0.184	0.883	0.005	0.266	0.266	0.445	1.309
30%	-0.319	0.198	0.376	0.227	0.879	0.008	0.242	0.242	0.482	1.271
40%	-0.309	0.191	0.363	0.247	0.881	0.010	0.232	0.232	0.497	1.264
50%	-0.303	0.190	0.358	0.260	0.886	0.012	0.226	0.226	0.507	1.250

Source: author's calculations

Table 7 - Monte Carlo RIF autoregressive, sample size = 1,250. Autoregressive coefficients.

α	φ1					γ1				
	Bias	Std.	RMSE	Perc. 5%	Perc. 95%	Bias	Std.	RMSE	Perc. 5%	Perc. 95%
1%	-0.303	0.072	0.312	0.078	0.316	-0.115	0.171	0.206	0.128	0.681
5%	-0.257	0.041	0.260	0.176	0.310	-0.049	0.090	0.102	0.304	0.598
10%	-0.232	0.035	0.235	0.211	0.326	-0.026	0.066	0.071	0.364	0.581
20%	-0.206	0.030	0.208	0.244	0.344	-0.009	0.049	0.049	0.412	0.572
30%	-0.191	0.029	0.193	0.262	0.357	-0.001	0.041	0.041	0.430	0.567
40%	-0.183	0.028	0.185	0.272	0.363	0.002	0.038	0.038	0.440	0.565
50%	-0.180	0.027	0.183	0.276	0.366	0.003	0.036	0.036	0.443	0.563

Source: author's calculations

Table 8 - Monte Carlo RIF autoregressive, sample size = 1,250, Coefficients of x_t.

α	φ_2					γ_2				
	Bias	Std.	RMSE	Perc. 5%	Perc. 95%	Bias	Std.	RMSE	Perc. 5%	Perc. 95%
1%	-0.395	0.414	0.572	-0.194	1.139	-0.172	0.562	0.587	-0.151	1.672
5%	-0.431	0.228	0.488	0.071	0.821	-0.052	0.322	0.326	0.294	1.352
10%	-0.393	0.187	0.436	0.172	0.784	-0.014	0.255	0.255	0.432	1.273
20%	-0.348	0.161	0.384	0.260	0.784	0.007	0.205	0.205	0.550	1.218
30%	-0.324	0.151	0.357	0.301	0.796	0.012	0.186	0.186	0.579	1.193
40%	-0.311	0.145	0.343	0.322	0.796	0.011	0.175	0.175	0.593	1.172
50%	-0.307	0.144	0.339	0.325	0.796	0.011	0.172	0.173	0.600	1.162

Source: author's calculations

The data

Table 9 – Summary statistics.

	Mean	Std	Asymmetry	Kurtosis	Perc. 5%	Perc. 1%
Portfolio 1	0.614	2.742	0.682	3.536	-3.380	-6.148
Portfolio 2	0.442	2.042	0.372	2.590	-2.600	-5.076
Risk factors						
Duration	-0.120	2.766	0.839	4.888	-3.986	-7.036
D2 – Slope	0.110	4.479	-0.523	3.267	-7.454	-11.414
D3 - Convexity	0.037	2.153	-0.126	1.201	-3.624	-5.633
Agency	0.045	0.531	-0.354	5.563	-0.770	-1.208
Industrial	-0.002	1.278	0.121	2.970	-2.078	-3.193
Financial	0.040	1.343	0.185	1.920	-2.020	-3.290
Bank	0.011	1.318	0.071	2.303	-2.160	-3.366
Phone	-0.087	1.698	-0.146	1.326	-2.960	-4.300
Electric	0.026	1.560	-0.039	1.244	-2.591	-3.986
Utility	-0.025	2.045	-0.962	18.199	-3.138	-5.569
Ginnie MBS	-0.018	1.846	0.028	2.830	-2.798	-5.568
Fannie MBS	-0.067	1.632	0.398	32.222	-2.132	-4.608
Freddy MBS	-0.029	1.441	0.096	5.800	-2.170	-4.163
Supra	0.094	2.506	-0.047	5.470	-3.514	-7.853
Yankee	0.092	1.515	0.387	6.436	-2.316	-4.064
Aa	-0.009	0.964	-0.219	2.438	-1.510	-2.638
A	-0.010	1.125	-0.335	3.295	-1.740	-3.039
Baa	-0.079	1.455	-0.272	1.766	-2.634	-4.109
Ba	0.116	6.099	-0.378	3.960	-9.364	-18.032
B	0.054	9.091	-0.401	3.844	-14.408	-27.720
Caa	-0.560	12.480	-0.452	3.657	-21.842	-39.148
Term Spread	-0.001	1.487	0.036	2.162	-2.171	-3.630
Volatility	-4.045	68.808	-0.603	7.593	-109.468	-212.492
Ginnie prepay-elst	0.494	15.983	0.049	1.824	-25.080	-41.512
Fannie prepay-elst	-1.171	15.349	-0.730	48.724	-20.266	-55.057
Freddy prepay-elst	0.349	15.952	0.023	1.347	-25.470	-38.790
MBS 15 yr	0.014	1.015	0.368	7.317	-1.558	-2.601
Inflation	0.003	2.094	0.055	1.988	-3.472	-5.135
Indexes						
1-3y US Treasury	0.519	5.756	0.528	2.727	-7.799	-14.084
0-1y US Treasury	-0.028	0.755	1.110	4.821	-1.098	-1.577
Libor 3M	-0.132	0.826	0.748	5.573	-1.413	-2.311
1-3yr US Corp	0.917	5.387	0.213	1.943	-7.061	-13.943
U,S, MBS	0.962	14.187	-0.142	1.693	-22.824	-36.540
1-10 Year TIPS	0.848	17.736	0.025	1.422	-28.289	-43.614

Source: Wilshire Associates, Bloomberg and St. Louis FED.
Author's calculations

Models results

Table 10 - Portfolio 1, regression on indexes, VaR-UQR.

Variable	Coefficient	5%	95%	Signif.
(Intercept)	-3.695	-4.402	-2.908	*
1-3y US Treasury	0.040	-0.195	0.228	
0-1y US Treasury	-0.184	-0.521	0.075	
Libor 3M	-0.189	-0.441	0.140	
1-3yr US Corp	0.218	0.061	0.388	*
U,S, MBS	-0.025	-0.088	0.060	
1-10 Year TIPS	0.091	0.013	0.151	*
lagged portfolio	-0.142	-0.300	-0.001	*

Source: Bloomberg y FED de St. Louis.
Author's calculations
Significant at 10% (*).

Table 11 - Portfolio 1, regression on indexes, CAViaR.

Variable	Coefficient	Std	Signif.
(Intercept)	-0.598	0.103	*
1-3y US Treasury	0.057	0.036	
0-1y US Treasury	0.095	0.105	
Libor 3M	-0.055	0.043	
1-3yr US Corp	0.175	0.034	*
U,S, MBS	0.004	0.005	
1-10 Year TIPS	0.092	0.006	*
lagged portfolio	-0.264	0.063	*
Caviar lag	0.224	0.090	*

Source: Bloomberg y FED de St. Louis.
Author's calculations
Significant at 10% (*).

Table 12 - Portfolio 1, regression on risk factors, VaR -UQR.

Variable	Coefficient	5%	95%	Signif.
(Intercept)	-3.313	-3.886	-2.773	*
Duration	0.363	0.137	0.603	*
D2 – Slope	0.309	0.129	0.506	*
D3 – Convexity	0.605	0.260	0.914	*
Agency	0.553	-0.586	1.564	
Industrial	-1.476	-3.253	0.217	
Financial	0.377	-1.416	1.838	
Bank	-0.390	-1.311	0.656	
Phone	0.425	0.051	0.762	*
Electric	0.348	-0.361	1.184	
Utility	0.201	-0.416	0.776	

Ginnie MBS	-0.421	-0.817	-0.052	*
Fannie MBS	0.014	-0.669	0.771	
Freddy MBS	0.514	-0.433	1.289	
Supra	1.024	0.524	1.559	*
Yankee	0.423	-0.314	1.288	
Aa	-0.506	-2.465	1.186	
A	1.677	-0.907	4.488	
Baa	-0.781	-1.930	0.544	
Ba	0.202	-0.226	0.557	
B	-0.085	-0.255	0.118	
Caa	-0.022	-0.074	0.038	
Term Spread	0.575	-0.051	1.108	
Volatility	0.005	-0.006	0.021	
Ginnie prepay-elst	0.046	0.012	0.081	*
Fannie prepay-elst	0.025	-0.007	0.064	
Freddy prepay-elst	-0.031	-0.065	0.003	
MBS 15 yr	0.470	-0.126	1.065	
Inflation	-0.369	-0.632	-0.066	*
lagged portfolio	-0.318	-0.449	-0.163	*

Source: Wilshire Associates.
Author's calculations
Significant at 10% (*).

Table 13 - Portfolio 1, regression on risk factors, CAViaR.

Variable	Coefficient	Std	Signif.
(Intercept)	-1.524	0.467	*
Duration	0.445	0.174	*
D2 - Slope	0.305	0.122	*
D3 - Convexity	0.498	0.129	*
Agency	1.087	0.598	*
Industrial	-0.734	0.748	
Financial	1.051	1.425	
Bank	-0.869	0.608	
Phone	0.075	0.262	
Electric	0.148	0.760	
Utility	0.544	0.366	
Ginnie MBS	-0.040	0.243	
Fannie MBS	-0.424	0.822	
Freddy MBS	0.367	0.870	
Supra	0.782	0.283	*
Yankee	0.106	0.806	
Aa	0.032	1.274	
A	1.500	1.473	
Baa	-1.043	0.872	

Ba	0.178	0.190	
B	-0.074	0.074	
Caa	0.012	0.038	
Term Spread	0.424	0.346	
Volatility	0.006	0.012	
Ginnie prepay-elst	0.001	0.029	
Fannie prepay-elst	0.037	0.054	
Freddy prepay-elst	-0.020	0.038	
MBS 15 yr	0.045	0.337	
Inflation	-0.225	0.101	*
lagged portfolio	-0.334	0.071	*
Caviar lag	0.424	0.147	*

Source: Wilshire Associates.
Author's calculations
Significant at 10% (*).

Table 14 - Portfolio 2, regression on indexes, VaR-UQR.

Variable	Coefficient	5%	95%	Signif.
(Intercept)	-2.990	-3.506	-2.415	*
1-3y US Treasury	-0.106	-0.238	0.022	
0-1y US Treasury	-0.126	-0.344	0.078	
Libor 3M	-0.095	-0.327	0.331	
1-3yr US Corp	0.276	0.162	0.378	*
U,S, MBS	0.001	-0.019	0.033	
1-10 Year TIPS	0.083	0.041	0.102	*
lagged portfolio	-0.109	-0.196	-0.013	*

Source: Bloomberg y FED de St. Louis.
Author's calculations
Significant at 10% (*).

Table 15 - Portfolio 2, regression on indexes, CAViaR.

Variable	Coefficient	Std	Signif.
(Intercept)	-0.875	0.079	*
1-3y US Treasury	0.085	0.020	*
0-1y US Treasury	-0.018	0.085	
Libor 3M	-0.100	0.144	
1-3yr US Corp	0.059	0.017	*
U,S, MBS	0.001	0.005	
1-10 Year TIPS	0.063	0.007	*
lagged portfolio	-0.080	0.049	*
Caviar lag	0.053	0.063	

Source: Bloomberg y FED de St. Louis.
Author's calculations
Significant at 10% (*).

Table 16 - Portfolio 2, regression on risk factors, VaR-UQR.

Variable	Coefficient	5%	95%	Signif.
(Intercept)	-2.629	-3.031	-2.228	*
Duration	0.023	-0.118	0.180	
D2 – Slope	0.064	-0.040	0.198	
D3 – Convexity	0.282	0.146	0.467	*
Agency	0.256	-0.167	0.680	
Industrial	0.247	-0.608	0.868	
Financial	0.609	-0.216	1.307	
Bank	-0.656	-1.141	-0.172	*
Phone	0.061	-0.168	0.317	
Electric	-0.236	-0.706	0.246	
Utility	-0.038	-0.179	0.218	
Ginnie MBS	-0.257	-0.565	0.032	
Fannie MBS	-0.097	-0.628	0.314	
Freddy MBS	0.308	-0.126	0.821	
Supra	0.308	0.117	0.493	*
Yankee	-0.388	-0.675	-0.010	*
Aa	0.100	-0.938	1.150	
A	-0.428	-1.275	0.721	
Baa	0.671	-0.402	1.479	
Ba	0.098	-0.026	0.234	
B	-0.105	-0.193	-0.023	*
Caa	0.008	-0.025	0.043	
Term Spread	-0.284	-0.594	0.047	
Volatility	-0.002	-0.008	0.004	
Ginnie prepay-elst	0.013	-0.003	0.032	
Fannie prepay-elst	0.024	-0.004	0.055	
Freddy prepay-elst	-0.012	-0.039	0.010	
MBS 15 yr	0.140	-0.072	0.336	
Inflation	-0.122	-0.287	0.012	
lagged portfolio	-0.198	-0.304	-0.068	*

Source: Wilshire Associates.
Author's calculations
Significant at 10% (*).

Table 17 - Portfolio 2, regression on risk factors, CAViaR.

Variable	Coefficient	Std	Signif.
(Intercept)	-0.993	0.255	*
Duration	0.177	0.099	*
D2 – Slope	0.158	0.050	*
D3 - Convexity	0.331	0.093	*
Agency	-0.015	0.229	

Industrial	0.308	0.383	
Financial	0.593	0.351	*
Bank	-0.214	0.292	
Phone	0.036	0.122	
Electric	-0.357	0.246	
Utility	-0.047	0.077	
Ginnie MBS	-0.047	0.116	
Fannie MBS	-0.231	0.305	
Freddy MBS	0.138	0.287	
Supra	0.241	0.082	*
Yankee	-0.263	0.133	*
Aa	0.270	0.320	
A	-0.286	0.500	
Baa	0.366	0.281	
Ba	0.030	0.055	
B	-0.087	0.048	*
Caa	0.028	0.028	
Term Spread	-0.080	0.161	
Volatility	0.003	0.002	
Ginnie prepay-elst	0.001	0.014	
Fannie prepay-elst	0.049	0.020	*
Freddy prepay-elst	-0.028	0.017	
MBS 15 yr	0.056	0.151	
Inflation	-0.038	0.041	
lagged portfolio	-0.235	0.035	*
Caviar lag	0.510	0.091	*

Source: Wilshire Associates.
Author's calculations.
Significant at 10% (*).