Asymmetric Effects of Monetary Policy on the Colombian House Prices*

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Abstract

This article examines asymmetric effects of monetary policy on real estate price growth dynamics. We estimate a Markov Switching model using monthly data from 1994 to 2015 for the Colombian house prices' growth rates. Empirical results suggest that housing price growth has a larger magnitude decrease with a contractionary monetary shock in higher volatility periods than during calm ones, but is not as persistently as in less volatile ones. This suggests that monetary policy is more effective in terms of reducing housing price growths during crisis periods than those when economic conditions are more favorable.

Keywords: Markov switching model, real estate price, monetary policy.

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1 Introduction

The role of the housing market has attracted several authors in the last decade. House price growth has been studied in different countries, usually seeking to identify bubbles[1] or overindebtedness, in order to prevent other bubble burst such as the one occurred in the United States in 2008. Leamer (2007) established the importance of residential investment on the business cycle, and proposed that monetary policy should take into account property markets. In line with the previous statement, Mishkin (2007) describes six channels through which monetary policy affects house prices, and shows how changes in real estate value given by monetary shocks impact household consumption.

It is generally accepted that contractionary monetary policy (higher interest policy rate) has negative effects on house prices. Cheng & Jin (2013) argued that the implementation of a systematic monetary policy was useful to stabilize house prices in United States during the Volcker-Greenspan era. They showed that an increase in 50 basis points (0.5%) in the nominal interest rate, has an accumulated effect of 40% decrease in house prices after 20 quarters. Another study, done by Xu & Chen (2012) provided evidence that an expansionary monetary policy increased Chinese housing prices growth rates, obtaining similar results for the Australian case as well. On the other hand, Wadud et al. (2012) found that a contractionary monetary policy decreases housing activity, and increases house prices in short term, but with a contrary effect in the long term, which is consistent with the user cost approach.

Recently, asymmetric effects have been studied in some countries, with the aim of taking into account that different states[2] caused different house price reaction to monetary policy. Simo-Kengne et al. (2013) used a Markov-Switching VAR to analyze regime dependent impulse response functions with a contractionary shock, finding evidence of asymmetries in monetary policy transmission between bear and bull regimes. In addition, it is shown that an increase in T-bill interest rate in the bull market regime reduces house prices in all segments, however, during the bear market-regime the effect was larger. In a related study, Chowdhury & Maclennan (2014) found that a decrease of 100 basis points of the interest rate increases house prices in 59 basis points during a expansionary state, while the effect is of only 45 during recessionary periods. In line with these findings, Tsai (2013) showed that house prices have an asymmetric response to monetary

[1] See for example Hall & Sola (1999), de A. Ferreira (2006), Xiao & Tan (2006), Y. Lai & Jia (2009), Shi (2010), Phillips et al. (2012), Phillips et al. (2013). For the colombian case, see Gomez et al. (2013).

[2] Such as inflation targeting implementation, boom and bust periods, or bull and bear regimes.

policy in the United Kingdom: expansionary shocks tend to increase house prices but under contractionary policy, these do not react at all. They argue that this result is due to downward nominal price rigidities.

The goal of policymakers is usually to achieve both the highest economic growth and low inflation. Several studies focused on the monetary policy transmission from house markets to household consumption, looking for reinforced effects given by wealth or balance sheet channels; Giuliodori (2002) showed that house prices play an important role in the monetary policy transmission to consumption spending for United Kingdom, Finland and Sweden, while Aoki et al. (2002), Iacoviello & Minetti (2002; 2008) and Elbourne (2008) studied the housing channel for the United Kingdom and found evidence its existence. Specifically, the later assessed that only 12% to 15% of changes in consumption spending, after a tightening of monetary policy is due to the house channel, while Giuliodori (2002) attributed 60-70% of consumption fall to property market transmission.

In contrast with the previous, the Hong Kong Monetary Authority (2008) showed that monetary transmission to economic growth is small, but it has a large effect on inflation. China also presents evidence of wealth channel effects through different assets, specially residential ones, for which an appreciation of the housing assets (from an expansionary policy shock) increases household wealth (Koivu, 2012). Finally, Bjørnland & Jacobsen (2010) showed for the United States that, after a contractionary shock, stock prices fell immediately while house prices had a gradual response, nonetheless, finding that property had a larger impact on GDP and inflation.

In line with the arguments presented above, our main contribution is to study the effect of a contractionary monetary policy on the growth rate of house prices in Colombia, by showing the state changing patterns of the series. We estimate a Markov-Switching model and a regime dependent dynamic multiplier analysis for the scope using data on Colombian house prices, interest rates, income measured by the GDP, inflation, housing construction licenses and a housing cost construction index. It is worth to note that to our knowledge, there have not been any applications of this framework for Colombia or any other Latin American country.

The remainder of this paper is organized as follows. Section 2 provides a theoretical background about monetary policy effects on house markets, and we explain the model used to estimate the impact of monetary policy on house prices growth rates. The empirical methodology and econometric model are presented in section 3. Section 4 discusses the main findings of the study, and finally, section 5 presents some concluding remarks.

2 Theoretical Background

2.1 Monetary Policy Channels To Housing Market

Monetary policy effects on house prices and through them to the economy has been a topic of interest for monetary policymakers; Mishkin (2007) described how the changes done by the central bank to short term interest rates could affect the housing market and through it, the overall economy. He proposed six transmission channels classified into two main groups: direct and indirect ones. The first ones are effects of interest rates on *user cost of capital*, *expectations of future house price movements* and *housing supply*. The indirect channels consists on *wealth effects from house prices*, *balance sheet and credit effects on consumer spending* and *balance sheet and credit-channel effects on housing demand*.

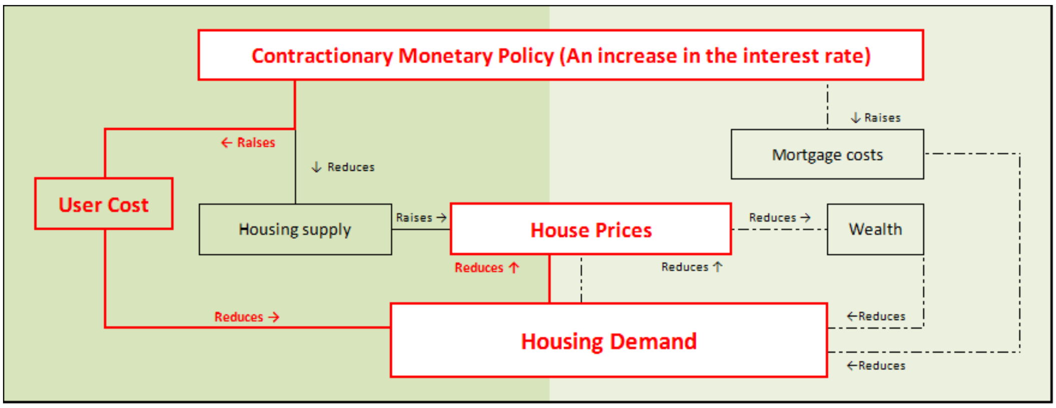


Figure 2.1: Monetary policy transmission channel to housing prices. Based on Wadud et al. (2012)

Figure 2.1 is based on the analysis done by Wadud et al. (2012). On the left hand side we observe direct channels while in the right-hand side we can see indirect ones.

2.1.1 User Cost Of Capital

The user cost of capital takes into account different factors that can affect the demand for a certain durable good (i.e. housing). It is represented as

$$uc_t = p_t^h \left[i_t - \pi_t - \frac{\Delta p_t^{h,e}}{p_t^h} \right]. \tag{2.1}$$

Where p_t^h is the house price, i_t is the nominal mortgage interest rate, π_t is the inflation,$p_t^{h,e}$ is the expected house price, and $\frac{\Delta p_t^{h,e}}{p_t^h} = \frac{p_{t+1}^{h,e} - p_t^h}{p_t^h}$ represent the capital gains. Mortgage interest rates are usually defined for long periods. These rates are expected long term rates. Monetary policy affects user cost of capital, through mortgage interest rates: when short term interest rises, long term ones also rise because of expectations. Re-expressing (2.1) gives the following relationship between the prices and the nominal mortgage interest rate:

$$p_t^h = \frac{uc_t + p_{t+1}^{h,e}}{1 + i_t - \pi_t}.$$

Then, a tightening in the monetary policy increases user cost and reduces house prices.

2.1.2 Expectations Of Future House Price Movements

In line with the previous argument, when monetary policy tightens, housing prices tend to decrease through the previous channel. When this happens, expectations of future house price movements decline causing an increase in user cost, making house prices to fall over time.

2.1.3 Housing Supply Channel

From the builder's perspective, short term interest rates are relevant for their financing purposes. Builders construct houses quickly (no more than 2 years), a reason that explains why these agents need to look for low short interest rates. An increase in policy interest rate implies higher costs of funding, reducing housing activity, and reducing housing supply, which implies an increase of prices.

2.1.4 Wealth Effects Of House Prices

When assets' prices increase, the return of household's portfolios is positively affected. Real estate is one of the multiple existing long term assets that compose household's portfolio. When monetary policy tightens, interest rates increase, and house prices decline, thus having a negative impact on the financial wealth of homeowners. This has the same effect on consumption by reducing the households' disposable income. Given that the house price volatility is not as high as it is for other asset prices (e.g. stock prices), changes caused by monetary policy usually have a longer effect on household's wealth.

2.1.5 Balance Sheet And Credit Channel On Consumption

The main problem that lenders face when making loans consists on the asymmetric information about borrowers. Adding collateral assets help to reduce informational problems for lenders because it decreases the losses to the lender if a borrower defaults: if we think in a house as a collateral for homeowners, then, when house price increases, the collateral asset price does as well. In effect, an expansive monetary policy usually lowers credit constraints for households, meaning a higher collateral that conducts to a higher probability to access the credit market, implying an increase in consumption.

2.1.6 Balance Sheet And Credit Channel On Housing Demand

Contractionary monetary policy can also reduce housing demand through a credit channel effect: when interest rates increase, nominal rates also rise, reducing cash flows. The latter effect decreases demand for houses. When cash flows are reduced, the availability of mortgages for households declines, thus making housing demand to decrease, which results in lower house prices.

In our study, we based the theoretical framework on the left-hand channel through the user cost of capital approach, which we present below.

2.2 Life-cycle Housing Model

See Appendix A.1 for a detailed walkthrough on demand side model calculations.

The Life-cycle model has been used from early eighties to explain behavior of households' expenses. In their seminal paper, Buckley & Ermisch (1982) developed a first example of a life-cycle model including housing expenditure. Their model incorporates a user cost approach to house price determination. This approach has been used since then by several authors, such as Meen (1990; 2001; 2002), Giussani & Hadjimatheou (1992; 1991), Breedon & Joyce (1993), Muellbauer & Murphy (1997), Pain & Westaway (1997), Holly & Jones (1997), Brown et al. (1997), Ashworth & Parker (1997) Meen & Andrew (1998), Gallin (2004; 2006), Skaarup & Bodker (2010),Qingyu (2010), Yan et al. (2010), Duca et al. (2011), Anundsen (2012), Auterson (2014).

Our model follows the life-cycle model proposed by Skaarup & Bodker (2010), combining it with the user cost specification of Meen (2001), as well as the monetary policy rule explained by Rincon et al. (2014).

2.2.1 Demand Side

The representative household maximizes its discounted utility function choosing between two goods: a composite consumption good (c_t) and housing services (h_t), discounted by the rate of time preference (θ). Lifetime utility can be written as:

$$max_{c,h}\ E_0 \sum_{t=0}^{\infty} \frac{1}{(1+\theta)^t} U(c_t, h_t) \tag{2.2}$$

The representative household maximizes the equation (2.2) subject to the following budget constraint:

$$A_t = (1+r)A_{t-1} + y_t - c_t - p_t^h h_t \tag{2.3}$$

Where A_t is real financial wealth. r is the constant real riskless return on the equilibrium asset portfolio (includes housing equity and bonds), y_t is real disposable income, and p_t^h is the real house price, while the price of c_t is numeriare. The present value of equation (2.3) is:

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} c_t = A_0 + \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} (y_t - p_t^h h_t)$$

Solving the household problem, we obtain the following marginal rate of substitution between composite consumption and housing services, that is equal to the relative prices.

$$\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = p_t^h \tag{2.4}$$

Equation (2.4) is an intratemporal substitution relationship. We can also find Euler's equations of intertemporal substitution of composite consumption and housing services,

$$\frac{U_c(c_t,h_t)}{U_c(c_{t+1},h_{t+1})} = \beta(1+r), \tag{2.5}$$

$$\frac{U_h(c_t,h_t)}{U_h(c_{t+1},h_{t+1})} = \beta(1+r)(1+\pi_t^h), \tag{2.6}$$

where π_t^h is the gross growth of housing prices and $\beta = \frac{1}{(1+\theta)}$. Using a specific CRRA utility function $\left(U(c_t,h_t) = \frac{\gamma c_t^{(1-\phi)} + (1-\gamma)h_t^{(1-\phi)}}{1-\phi}\right)$, the intratemporal relationship in logarithms can be written as,

$$log(p_t^h) = \phi[log(c_t) - log(uc_t) - log(H_t)] + log\left(\frac{1-\gamma}{\gamma}\right). \tag{2.7}$$

Inserting Euler equations (2.5,2.6) in the budget constraint expression and solving for present value we can find,

$$c_t = \mu(A_t + \widehat{y}_t - \nu_t p_t^h h_t). \tag{2.8}$$

We also take into account the Meen's expression of user cost,

$$uc_t = p_t^h \left[i_t - \pi_t - \frac{\Delta p_t^{h,e}}{p_t^h} \right], \tag{2.9}$$

where i_t is the nominal mortgage interest rate, π_t is general inflation rate and the last term is the expected real capital gain $CG_t = \frac{\Delta p_t^{h,e}}{p_t^h}$.

We take logarithms and use a Taylor expansion of (2.8), and replacing it and (2.9) on (2.7) we find the demand side house price equation.

2.2.2 Supply Side

Firm's profits are given by the value of housing sales $p_t^h H_t$, minus the cost of investment in housing by firms, that is divided in direct costs $C_t^I I_t$ and some convex costs of investment $C(I_t,H_t)$. The representative firm chooses the optimal amount of investment by maximizing the net present value of profits,

$$max_{H,I}\ \Pi = \sum_{t=0}^{\infty} (1+r)^{-t} [p_t^h H_t - C_t^I I_t - C(I_t,H_t)], \tag{2.10}$$

subject to the housing stock evolution constraint,

$$H_{t+1} = (1+\delta)H_t + I_t. \tag{2.11}$$

Solving the firm's problem we find that firms invest until the price of housing equals the total cost of investment,and we obtain the supply investment equation,

$$log\left(\frac{I_t}{H_t}\right) = \frac{1}{\phi}\left[\rho + \frac{\omega}{\omega-1}log(p_t^H) - \frac{1}{\omega-1}log(C_t^I)\right] \tag{2.12}$$

This expression can be rearranged to show that house price from the supply side view depends positively on cost of investment in housing by firms.

2.2.3 Central Bank

The way how monetary policy influences the house price is through user cost channel. Where the central bank increases their interest rates (a contractionary policy) nominal interest rates increase. The monetary policy followed by the central bank can be written as,

$$i_t = \rho_s i_{t-1} + (1-\rho_s)[\rho_i \bar{i} + \rho_\pi(\pi_t - \bar{\pi})] + \upsilon_t. \tag{2.13}$$

Equation (2.13) is a version of the FISCO model monetary policy equation for Colombia (Rincon et al., 2014). This shows that the central bank responds to deviation in inflation from target $\bar{\pi}$. It also is adjusted in a proportion ρ_s to lagged value. Using a Markov Switching Taylor rule as proposed by Rabanal (2004), we can rewrite (2.13) as,

$$i_t = \rho_s(s_t)i_{t-1} + (1-\rho_s(s_t))[\rho_i(s_t)\bar{i} + \rho_\pi(s_t)(\pi_t - \bar{\pi})] + \upsilon_t. \tag{2.14}$$

Adding this rule, the model presents a more general framework which allows the economy to be at different states over time on its equilibrium form.

2.2.4 Solving the Model

Using supply and demand equations obtained from each maximization problem, and replacing the interest rate of the user cost by the monetary policy rule, we find the following price equation, :

$$\begin{aligned} log(p_t^h) = \beta_0(S_t) - \beta_i(S_t)i_{t-1} + \beta_\pi(S_t)\pi_t + \beta_A log(A_t) + \\ \beta_y log(\hat{y}_t) - \beta_I log(I_t) - \beta_C log(C_t^I) - \beta_\Delta CG_t \end{aligned} \tag{2.15}$$

Notice that in equilibrium the demand and supply housing price are equaled ($p_t^h = p_t^H$). $\hat{y}_t$ refers to the discounted future income flows. Investment costs increase housing prices, as a transmission of builders' costs to households. $\beta_0(s_t)$ is a collection of parameters that are constant in each state. We can rearrange equation 2.15 as a Markov Switching econometric model, expressing $\tilde{x} \equiv log(x)$ for simplification,

$$\tilde{p_t^h} = \beta_0(S_t) + \beta_1(S_t)i_{t-1} + \beta_2(S_t)\pi_t + \beta_3\tilde{A_t} + \beta_4\tilde{\hat{y_t}} + \beta_5\tilde{I_t^H} + \beta_6\tilde{C_t^I} + \beta_7 CG_t + \upsilon_t \tag{2.16}$$

With υ_t $i.i.d$ $N(0, \sigma^2(s_t))$. Equation (2.16) is in log-levels, then we can express it in growth rates, finding the following relationship:

$$gp_t^h = \gamma_0(s_t) + \gamma_1(s_t)\Delta i_{t-1} + \gamma_2(s_t)\Delta\pi_t + \gamma_3 gA_t + \gamma_4 g\hat{y}_t + \gamma_5 gI_t^H + \gamma_6 gC_t^I + \gamma_7\Delta CG_t + \varepsilon_t \tag{2.17}$$

where $CG_t = \frac{\Delta p_t^{h,e}}{p_t^h}$, left side $g's$ express growth rate of the mentioned variable, and Δ means first difference.

3 Empirical Methodology

This section presents the econometric model used to describe the House prices' regimes as a function of its own lagged values, as well as other covariates. First we present the model, the regime transition dynamics and finally, the regime switching model specification for these prices.

3.1 Markov Switching Models.

Hamilton (1994) built a theoretical framework for time series that exhibited different behaviors over time. These changes are mainly explained by market events, wars or permanent shocks in the structure across the different economic sectors that these series represent. Here, we present the state-dependent dynamics for the data generating process of these variables.

Let y_t be a random variable over time t which depends on multiple states $1, \ldots, N$. Then, the regime-switching data generating process for this variable can be described as:

$$y_t = \mu_{s_t} + \sigma_{s_t}\varepsilon_t \tag{3.1}$$

$$\mu_{s_t} = \beta_{s_t}X_t' \tag{3.2}$$

$$\varepsilon_t \sim N(0,1) \tag{3.3}$$

Where $s_t = 1, \ldots, N$ is a discrete random variable denoting the current state of the process.

In our application we have $N = 2$ states, one which will characterize the "normal" economic dynamics, and a "high-volatility" one which will capture the process dynamics when there is a high market volatility and potential crises. X_t is an explanatory regime-switching variables vector of order k, which may contain exogenous variables or lags of y_t, μ_{s_t} is the conditional mean of y_t given an information set for the s state in the time period t and ε_t is an innovation term. The parameter σ_{s_t} is also state-dependent, thus allowing to model different homoscedastic variances in the error term across regimes. Finally, β_{s_t} correspond to a parameter vector which depends on the system's state at the time t[3].

Furthermore, it is worth noting that the non linearity is given by the discrete variable s_t, which governs the regime switching dynamics in the following fashion: every time it changes it's value, i.e from 1 to 2, the parameters governing the data generating process of y_t will be different, thus showing a regime transition.

This model can typify transitory or permanent changes on the time series' generating process. It is worth noting that if these changes exist and are not modeled properly, then the information provided by the time series could be biased and as a consequence, it would be necessary to consider only observations after the given change or use dummy variables to model these different dynamics.

However, this approach implies doing arbitrary assumptions because the real dates in which the events happened may not be known or there may exist persistence when these changes happen. As a result of this, there is not a clear way to properly establish the right choice of dummy variables to be included in the model, thus leading to incorrect specifications that may lead to misguided conclusions.

In our empirical application, we use the house prices monthly growth rates and the first-difference of the 90-days Certificate of Deposit (DTF for its acronym in spanish) rate as a proxy of the monetary policy variable. House prices are known to have been influenced by two different economic settings that made evident changes on its data generating process: one of "normal" economic conditions and one displaying market nervousness. Moreover, these regimes are characterized by high and low volatility periods, which can be associated to crises or deep activity on the housing market.

[3] However, as some exogenous variables may not be regime-switching, this particular case would have a value of $\beta^l = \beta_{l1}, \beta_{l2}, \dots, \beta_{lN}$ for the variable in position l on the X_t vector.

3.2 Transition Among Regimes And Estimation.

Given what the monetary policy framework suggests, it is important to properly explain how Markov Switching Models characterize the regime changes through the discrete variable s_t; here, we show the transition dynamics of these models. Markov Switching models are based on first order Markov Chains and mixture distributions which allows to compute likelihood functions given by discrete changes on regimes switching according to transition probability matrices.

3.2.1 Transition Probabilities.

Let p_{ij} be the probability that s_t is on regime j given that it was on regime i ($P\{s_t = j|s_{t-1} = i\}$); as the conditional suggests, this probability depends only on the value the state variable s took on the previous period, a property derived from the first order Markov Chains which allows the model transition among regimes without many additional assumptions.

These probabilities are collected in a Transition matrix P as shown below:

$$P = \begin{bmatrix} p_{1,1} & p_{2,1} & \cdots & p_{N,1} \\ p_{1,2} & p_{2,2} & \cdots & p_{N,2} \\ \vdots & \vdots & \vdots & \vdots \\ p_{1,N} & p_{2,n} & \cdots & p_{N,N} \end{bmatrix} \tag{3.4}$$

Each column on this matrix sums to unity and shows the probability that the process switches to regime j from every possible regime i, as was explained before. This matrix allows to compute the filtered probabilities which shows how likely is that y_t is in regime j given the information at date ($\mathscr{Y}_t = \mathscr{Y}_{t-1}$ given the markov-chain properties) and a vector of model parameters θ. We denote this with the $N \times 1$ vector $\xi_{t|t} = P\{s_t = j|\mathscr{Y}_t; \theta\}$.

(3.4) allows to model changes on the filter probabilities when new information arrives to the system, thus characterizing a regime transition given that $\xi_{t|t}$ takes into account that when t changes to $t+1$. A forecasting filter for these probabilities is calculated by weighting $\xi_{t|t}$ by their associated values on P and added accordingly as it will be shown below.

3.2.2 Model Estimation

Hamilton (1994) proves that a likelihood function can be computed based on the calculation of a forecasting filter and mixture distributions that provide different dynamics for each regime. We briefly show the procedure to compute the likelihood function as well as the associated filter in this subsection.

Let η_t be a density function vector which contains the calculated density for each regime (i.e normal distribution) based on the sample information and a parameter vector $\theta^\star$:

$$\eta_t = \begin{bmatrix} f\left(y_t | s_t = 1, \mathscr{Y}_t, \theta^\star\right) \\ f\left(y_t | s_t = 2, \mathscr{Y}_t, \theta^\star\right) \\ \vdots \\ f\left(y_t | s_t = N, \mathscr{Y}_t, \theta^\star\right) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(\frac{-(y_t - x_t'\beta_1)^2}{2\sigma_1^2}\right) \\ \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(\frac{-(y_t - x_t'\beta_2)^2}{2\sigma_2^2}\right) \\ \vdots \\ \frac{1}{\sqrt{2\pi}\sigma_N} \exp\left(\frac{-(y_t - x_t'\beta_N)^2}{2\sigma_N^2}\right) \end{bmatrix} \tag{3.5}$$

where $y_t - x_t'\beta_j$ is the model residual when the process is in the regime $j, \theta^\star = [\beta_1, \beta_2, \ \ldots, \beta_N, \sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2]$ or more specifically, if we denote c_j the constant term associated with regime j, γ_{lj} the coefficient associated with the exogenous variable k on regime j, and $\phi_{l,j}$ the parameter associated to the lag l of y_t on the regime j, we get the following vector for all $j = 1, \ldots, N$ and $l = 1, \ldots, p$

$$\begin{aligned} \theta^\star = &[c_1, \gamma_{11}, \ldots, \gamma_{K1}\phi_{1,1}, \ldots, \phi_{p,1}, c_2, \gamma_{12}, \ldots, \gamma_{K2}, \phi_{1,2}, \ldots, \\ &\phi_{p,2}, \ldots, c_N, \gamma_{1N}, \ldots, \gamma_{KN}, \phi_{1,N}, \ldots, \phi_{p,N}, \sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2]. \end{aligned}$$

Given η_t, a set of parameters $\theta = [\theta^\star, p_{1,1}, p_{1,2}, \ldots, p_{1,N}, p_{2,1}, p_{2,2}, \ldots, p_{2,N}, \ldots, p_{N,1}, \ p_{N,2}, \cdots p_{N,N}]$ and a starting value for $\xi_{1|0}$, it is possible to compute the following filter for all $t = 1, \ldots, T$:

$$\hat{\xi}_{t|t} = \frac{\left(\hat{\xi}_{t|t-1} \odot \eta_t\right)}{1'\left(\hat{\xi}_{t|t-1} \odot \eta_t\right)} \tag{3.6}$$

$$\hat{\xi}_{t+1|t} = P \cdot \hat{\xi}_{t|t} \tag{3.7}$$

Then, the log-likelihood function is simply:

$$\mathscr{L}(\theta) = \sum_{t=1}^{T} \log f\left(y_t | \mathscr{Y}_{t-1}, \theta\right) \tag{3.8}$$

$$= \sum_{t=1}^{T} \log \left[1'\left(\hat{\xi}_{t|t-1} \odot \eta_t\right)\right] \tag{3.9}$$

It is worth to note that $1'\left(\hat{\xi}_{t|t-1} \odot \eta_t\right)$ is the mean of the density functions vector weighted by their respective filter probability forecasts. This can be seen as mixture likelihood function as was indicated before. So, for a given set of initial parameters θ_0, equation (3.8) can be maximized by conventional procedures[4], calculating the iterative process described in (3.6) for all t and then computing the corresponding log sum of weighted means. In addition, Xie et al. (2008) shows that these estimators are consistent and asymptotically normal, a property commonly known for maximum likelihood estimators.

3.3 Regime-Specific Dynamic Multipliers in the Markov-Switching Models Framework.

Ehrmann et al. (2003) pointed out that regime specific impulse response functions are valid under the assumption that the regimes are long standing, and can be computed following Krolzig (1997) approach. However, our variable of interest is i_t, which is assumed as exogenous in our model. In this case, Hu & Shin (2014) proposed the calculation of dynamic multipliers. Combining both approaches, we briefly summarize the methodology used to calculate these functions on the Markov Switching Autoregressive with Exogenous Variables (MS-ARX) model framework.

First, we begin with an MS-ARX model for the regime j with $(K = 1)$ exogenous variable and no intercept (without loss of generality) which includes $m = 0,\ 1,\ 2, \ldots,\ M$ lags, as shown below:

$$y_t = \phi_{1j}\, y_{t-1} + \ldots + \phi_{pj}\, y_{t-p} + \gamma_{1j,0}\, x_t + \gamma_{1j,1}\, x_{t-1} + \ldots + \gamma_{1j,M}\, x_{t-M} + \sigma_j \varepsilon_t \tag{3.10}$$

From this model, we can compute a regime specific ARX(1) representation, following Lütkepohl (2005) generalized VARX form, when there is only one variable present. In such case, the representation is given as follows:

$$Y_t = \boldsymbol{A}_j Y_{t-1} + \boldsymbol{B}_j x_t + \sigma_j \mathscr{E}_t \tag{3.11}$$

[4]Some of these procedures are the EM algorithm or linear-constrained methods. See Van Norden & Vigfusson (1996) and Hamilton (1990) for a detailed review.

where:

$$Y_t \equiv \begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-p+1} \\ x_t \\ x_{t-1} \\ \vdots \\ x_{t-M+1} \end{bmatrix}_{Kp\times 1} ; \mathscr{E}_t \equiv \begin{bmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{p+M\times 1} ;$$

$$\boldsymbol{A}_j \equiv \begin{bmatrix} \phi_{1j} & \dots & \phi_{p-1j} & \phi_{pj} & | & \gamma_{1j,1} & \dots & \gamma_{1j,M-1} & \gamma_{1j,M} \\ 1 & \dots & 0 & 0 & | & 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & 0 & | & 0 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & | & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & 0 & | & 0 & \dots & 0 & 0 \\ - & - & - & - & - & - & - & - & - \\ 0 & \dots & 0 & 0 & | & 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & 0 & | & 1 & \dots & 0 & 0 \\ 0 & \dots & 0 & 0 & | & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & 0 & | & 0 & \dots & 1 & 0 \end{bmatrix}_{p+M\times p+M} ;$$

$$\boldsymbol{B}_j \equiv [\underbrace{\gamma_{1,j,0}\ 0\ \dots\ 0}_{1\times p}\ \underbrace{1\ 0\ \dots\ 0}_{1\times M}]' \tag{3.12}$$

Successive substitution for lagged Y_t's gives

$$Y_t = (\boldsymbol{A}_j)^h Y_{t-h} + \sum_{i=0}^{h-1} (\boldsymbol{A}_j)^i \boldsymbol{B}_j x_{t-i} + \sum_{i=0}^{h-1} (\boldsymbol{A}_j)^i \mathscr{E}_{t-i} \tag{3.13}$$

Then, premultiplying (3.13) by $J \equiv [1\ 0 \dots\ 0]_{1\times(p+M)}$ gives the following expression:

$$y_{t+h} = J(\boldsymbol{A}_j)^h Y_t + \sum_{i=0}^{h-1} J(\boldsymbol{A}_j)^i \boldsymbol{B}_j x_{t+h-i} + \sum_{i=0}^{h-1} J(\boldsymbol{A}_j)^i J' \varepsilon_{t+h-i}$$

where $\mathscr{E}_t = J'J\mathscr{E}_t = J'\varepsilon t$ is used. Then, the optimal forecast given the information at time t and $x_{t+1}, x_{t+2}, \dots, x_{t+h}$ is simply

$$y_t(h|x) = J(\boldsymbol{A}_j)^h Y_t + \sum_{i=0}^{h-1} J(\boldsymbol{A}_j)^i \boldsymbol{B}_j x_{t+h-i}.$$

Following Lütkepohl (2005)'s impulse response function definition, it can be easily seen that the dynamic multiplier D for regime j on horizon (h) can be computed as:

$$D_j^{(h)} = J\left(\boldsymbol{A}_j\right)^h \boldsymbol{B}_j \tag{3.14}$$

This expression allows to find the dynamic effect of any exogenous variable (x_t) involved in the model over the dependent (y_t) one for any horizon desired by the researcher.

We proceed to our empirical assessment applied to the Colombian case. Taking up equation (2.17),

$$gp_t^h = \gamma_0(s_t) + \gamma_1(s_t)\Delta i_{t-1} + \gamma_2(s_t)\Delta\pi_t + \gamma_3 gA_t + \gamma_4 g\hat{y}_t + \gamma_5 gI_t^H + \gamma_6 gC_t^I + \gamma_7 \Delta CG_t + \varepsilon_t,$$

Where $Y_t = y_t + A_t$, this means that we use GDP as a proxy of income plus asset wealth of household. L_t is the approved squared meters for construction as a proxy of housing investment. CG_t is the expected capital gain, which is approximated in our model by the lags of house price growth. This implies that we suppose adaptive expectations of households for the house price. We use this approach given that Chow (1989) finds that "*an asset pricing model with adaptive expectations outperforms one with rational ones in accounting for observed movements in U.S. stock prices*" (Gelain & Lansing, 2014), as well as Bover (2010) who found that these expectations best fit house price returns.

Typically, as shown by Becerra & Melo Velandia (2009), monetary policy takes approximately three months to complete its transmission process to the interest rates. Thus, we will estimate the model with a total of $p = 3$ lags to take into account the adjustment of CG_t in response to changes in the monetary policy lags.

For the growth of house prices p_t^h we use the monthly growth rate of house price index obtained from the *Departamento Nacional de Planeación* (DNP). For the lagged policy rate i_{t-1}, we use the 90-days Certificate of Deposit rate (DTF). Inflation rate π_t, is the year-to-year change in the Colombian Consumer Price Index (CPI). DTF and inflation rates were obtained from the Colombian Central Bank. Taking the above assumptions into account, we rewrite equation (2.17) as,

$$gp_t^h = c_{s_t} + \widehat{\gamma_{\Delta i,s_t}}\Delta i_{t-1} + \widehat{\gamma_{\Delta\pi,s_t}}\Delta\pi_t + \widehat{\gamma_Y} gY_t + \widehat{\gamma_L} gL_t + \widehat{\gamma_C} gC_t^I + \sum_{p=1}^{P} \widehat{\phi_{p,s_t}} gp_{t-p}^h + \widehat{\sigma_{\xi,s_t}}\varepsilon_t. \tag{3.15}$$

Here we have intercept and variance switching parameters. Both, inflation and monetary policy rate, have switching coefficients, as well the lagged dependent ones. Then we have to estimate $3+p+1$ switching parameters, and three non regime-switching ones.

4 Results

This section presents some stylized facts as well as the model estimated for the house prices' growth rates, given some unit root concerns which will be discussed below.

4.1 Some Stylized facts and Unit Roots

Our dataset is composed by several indicators which were taken and/or constructed from different sources; in addition, we used temporal disaggregation methods to estimate them on a monthly frequency when relevant[5]. The sample ranges from January 1994 to December 2015. Figure 4.1 shows the evolution since April 1995 of inflation, Monetary Policy Rate (MPR) and the interest rate of 90-day certificate of deposit (DTF). These three series have had a decreasing trend, which can be associated to the response of the monetary policy rates to changes in the inflationary variables: given the decreasing layout of the inflation rates, the MPR and DTF should have this a similar pattern as well (Julio, 2006).

[5]See Appendix C for more details on this concern.

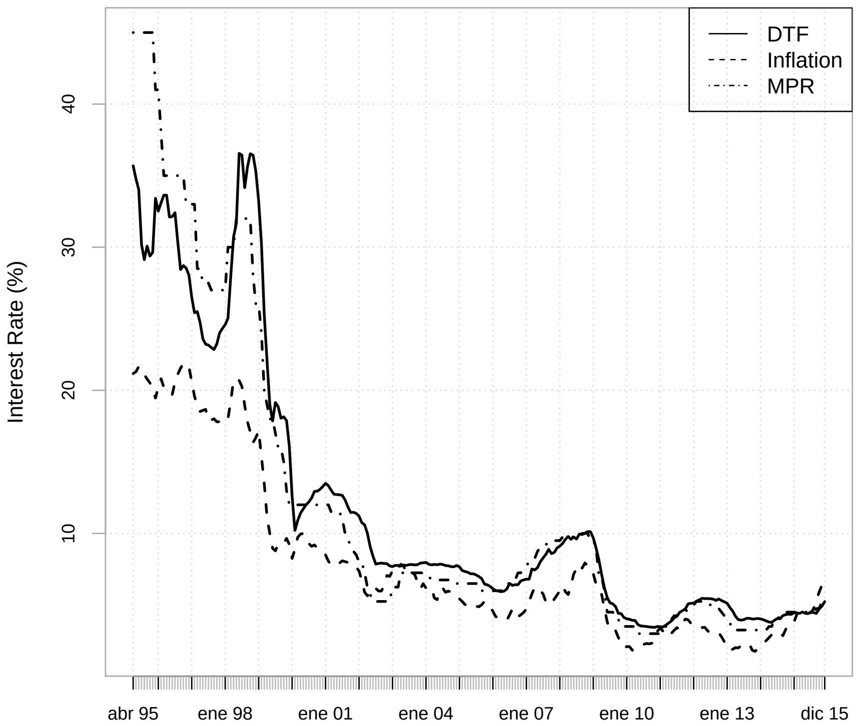


Figure 4.1: DTF, Inflation and MPR rates .

As presented in the theoretical approximation, equation (2.14) shows a Markov Switching Taylor rule. Figures 4.2 and 4.3 presents some evidence about how this class of Taylor rule could be relevant to be studied in the monetary transmission framework present in the Colombian economy.

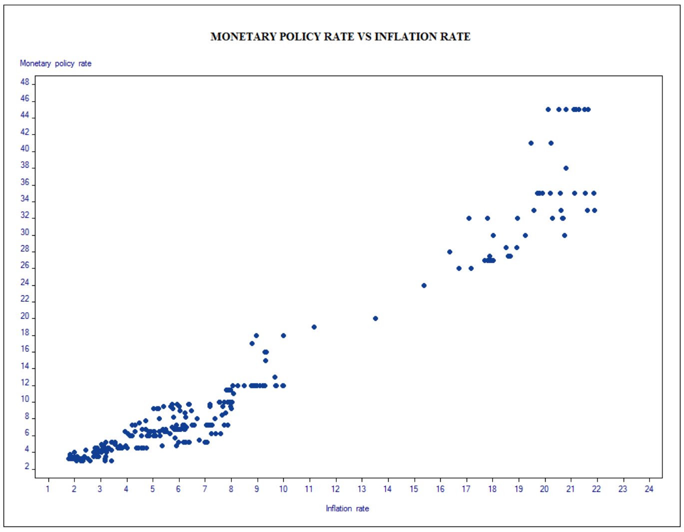


Figure 4.2: MPR and Inflation relation

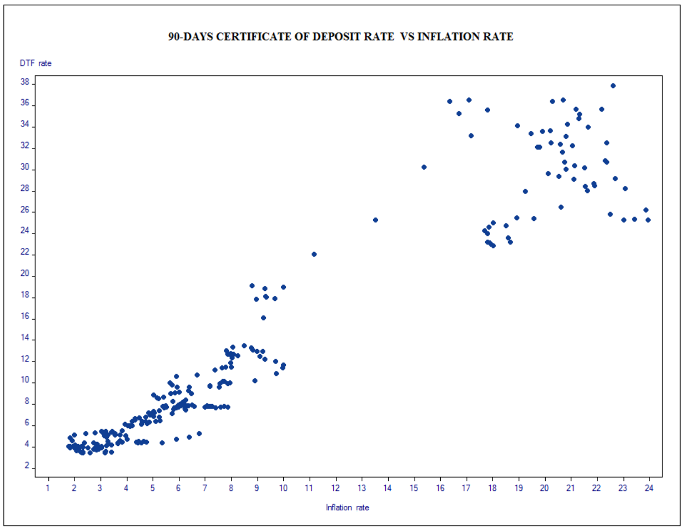


Figure 4.3: DTF and Inflation relation

Both Figures point out that the inflation is directly related with the monetary policy rates. However, the data seems to be grouped in two different mass points. This is explained by different monetary policy regimes: since the first decade of the XXI century, Colombia changed its monetary policy rule to inflation targeting. The presence of apparently different regimes is precisely what equation (2.14) allows to model.

In our empirical estimation we use DTF as an approximation of the monetary policy interest rate, given its similarity to the former at an aggregate level, as well as some statistical advantages that this rate presents, such as the fact that it has more variability over time; in addition, it is worth to notice that this rate responds directly and in a very similar way to changes in the monetary policy rates, an advantage that allows us to capture the effect of monetary policy on the housing prices.

Given the apparent existence of two different regimes present in the data, we perform our parameter estimations using $N = 2$ regimes. However, the presence of unit root is a recurring concern in the MS framework. In response to this, we performed some tests for the variables involved in our specification. Table 1 shows unit root tests for level variables, and for the input ones for the final model. Taking into account that house prices, monetary policy rate and inflation are regime dependent, we used a Markov Switching Unit Root (MSUR) test[6] to properly identify whether these variables have unit root on any of their regimes.

[6]See Appendix B for details on procedure description of a MSUR.

Markov-Switching ADF Test				
	State 1		**State 2**	
	T-Stat	**P-value**	**T-Stat**	**P-value**
House price (p_t^h)	-2.2162	0.3900	-2.7418	0.0105
Interest rate (i_t)	-2.1626	0.6135	-5.003518	0.0000
Inflation rate (π_t)	-1.8617	0.7727	-1.2201	0.2948
House price growth (gp_t^h)	-2.7189	0.0167	-4.1024	0.0000
Interest monthly change (Δi_t)	-3.1001	0.0495	-6.2428	0.0439
Inflation monthly change ($\Delta\pi_t$)	-4.7257	0.0027	-2.7418	0.0000

Table 1: MSUR test. Bootstrapping with 2000 replicas.

From Table 1 we conclude that all level variables are not stationary at least in one state (house price and interest rate), while all variables in growth rates or first-differences are stationary in both regimes at 5% level.

In the case of non regime-switching variables, we performed the conventional Unit Root tests[7], which are presented in the Table 2.

We can conclude that all variables in levels are not stationary while monthly growth rates do not display presence of unit roots.

Unit Root Tests					
Variable	**ADF**	**PP**	**KPSS μ**	**KPSS τ**	**Z-A**
GDP (Y_t)	UR	UR	NS	NS	NUR
Licenses (L_t)	UR	N.UR	NS	NS	N.UR
Cost (C_t)	UR	UR	S	NS	UR
GDP growth (gY_t)	N.UR	N.UR	S	S	N.UR
Licenses growth (gL_t)	N.UR	N.UR	S	S	N.UR
Cost growth (gC_t^I)	UR	N.UR	S	S	N.UR

UR = Unit root, N.UR = No Unit Root, S = Stationary, NS = Non-stationary. Conventions were used because of the difference in the null hypothesis proposed on each test

Table 2: Unit root tests at 5% significance.

4.2 Parameter Estimates

All of our variables in the model satisfy the stationary requirement for the Markov-Switching model, as was shown before. Table 3 shows the model estimation of 3.15 under the theoretical specification shown previously[8].

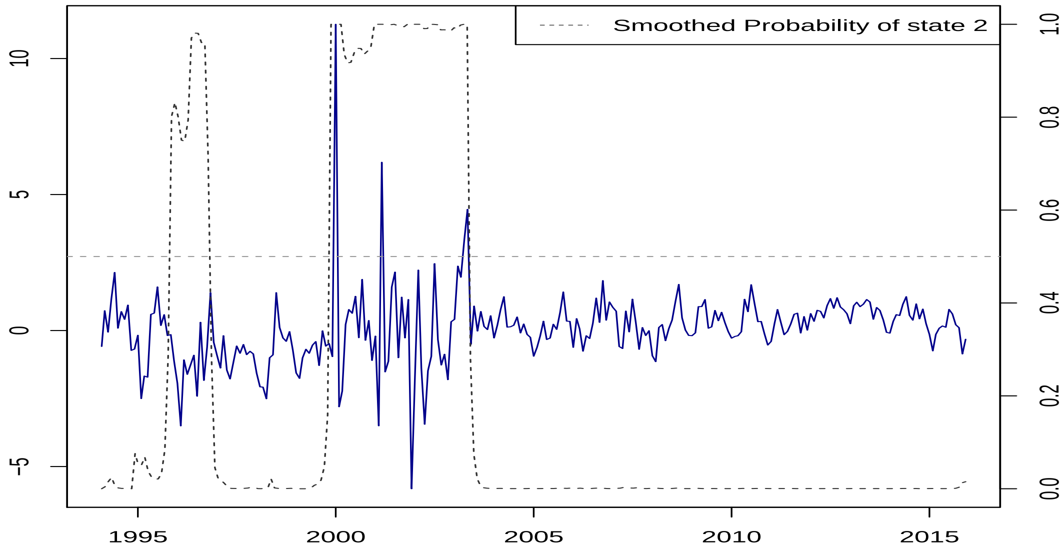
[7] We performed the tests presented in Dickey & Fuller (1979), Phillips & Perron (1986), Elliott et al. (1996) (ERS DF-GLS), Kwiatkowski et al. (1991) with intercept (KPSS μ) and trend (KPSS τ) and Zivot & Andrews (1992).

[8] In the Appendix D we present a table containing the full parameter estimation detail, including the standard errors.

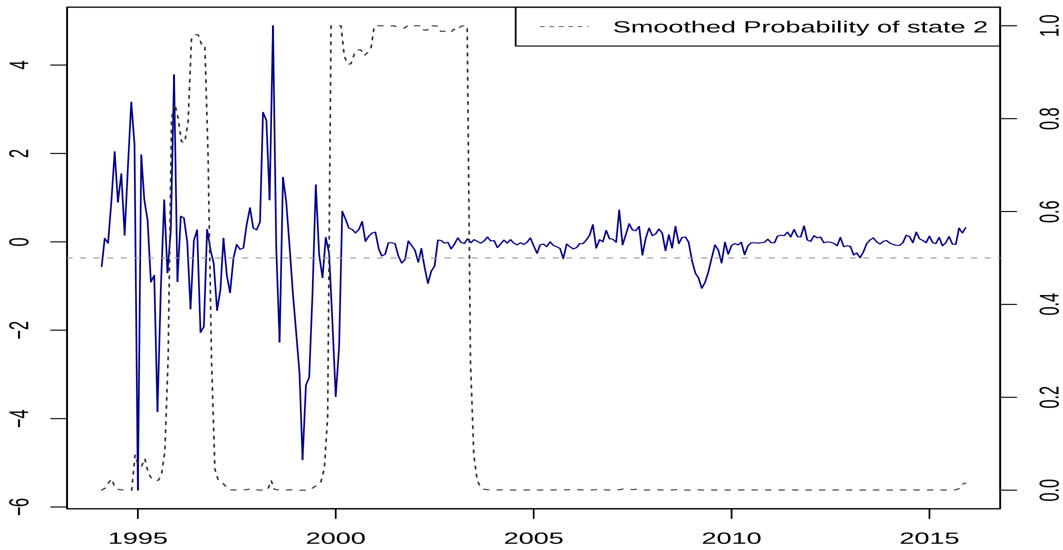
House Price Growth Model Estimation				
	Regime 1		Regime 2	
	Parameter	**P-Value**	**Parameter**	**P-Value**
c_{s_t}	0.0177	0.7146	-0.2754	0.4092
$\widehat{\gamma_{\Delta i,s_t}}$	-0.0178	0.7234	-0.9312	0.0366
$\widehat{\gamma_{\Delta \pi,s_t}}$	-0.0545	0.6595	-2.1608	0.0155
$\widehat{\phi_{1,s_t}}$	0.4850	0.0000	-0.0948	0.4493
$\widehat{\phi_{2,s_t}}$	0.2132	0.0031	0.0472	0.7176
$\widehat{\phi_{3,s_t}}$	-0.0850	0.1876	0.1312	0.3184
No Regime-Switching Parameters				
	Parameter	**P-Value**		
$\widehat{\gamma_Y}$	-0.0001	0.9758		
$\widehat{\gamma_L}$	0.0464	0.0003		
$\widehat{\gamma_C}$	0.1595	0.0880		
Log Likelihood:		-325.4369		
Unconditional Probabilities:			0.7948	0.2051
Transition Matrix and Regime specific error variances				
	1	2		
1	0.9881	0.0461		
2	0.0119	0.9539		
$\widehat{\sigma^2_{\varepsilon,s_t}}$	0.3678	5.1365		

Table 3: Parameter Estimates.

In the first regime, the most frequent one and thus the one with regular policy and economic behavior, we find that the DTF rate has a negative yet not statistically significant effect over the rate of growth of house prices. However, in the second one, we find a negative and significant relationship between these variables. This result leads us to conclude that monetary policy is effective in certain scenarios, where volatility is higher as shown by the $\widehat{\sigma}^2_{\varepsilon_t,2}$ parameter.



(a) Monthly House Prices Growth Rates.



(b) ΔDTF Rates.

Figure 4.4: Smoothed Probabilities on regime 2 versus Monthly House Prices Growth and DTF Rates.

In addition, Figure 4.4a seems to confirm this, because house prices display sharper movements and because of the inflation targeting policy, the DTF rate exhibits more modest movements during these periods, as shown by the Figure 4.4b. Higher housing price volatility is associated with a period spanned between 2000 and 2004 which is characterized by a paralysis in mortgage disbursements and recession in building area (ANIF, 2011), situations that make the real state market unstable, with prices moving around a stationary path, but with big ups and downs.

In line with the previous, autoregressive coefficients in each state confirm this hypothesis, as we can see that in the first state (regular state) is characterized by higher parameters, which means more persistence, while in the high volatility state, autoregressive parameters are near to zero, and are not significant. In the latter case (regime 2), a 100 basis points positive shock done by the central bank can potentially reduce the growth of such prices contemporaneously by roughly 93 basis points.

In line with the previous argument, we find that inflation has a negative influence over the house prices' growth rates in both regimes, being more severe (and significant) in the high volatility regime. This result is not surprising, since other authors such as Wadud et al. (2012) and Cheng & Jin (2013) have found a consistent inverse relationship between inflation rates and house prices.

4.3 Regime-Specific Dynamic Multipliers Estimation

So far, we have analyzed the parameters governing the contemporary effect of the central bank policy over the house prices. Nonetheless, these prices have regime specific dynamics that should be taken into account in order to properly estimate their true response to monetary policy shocks. To achieve this, regime-specific dynamic multipliers can be estimated under the assumption that each regime is long lasting (Ehrmann et al., 2003), the procedure to perform this estimation is described in section 3.3.

As can be seen in Figure 4.4, regime 1 duration[9] spans across very long time periods (more than 8 years), while the second one had a smaller yet significant duration of roughly 4 year on its longest timespan, which is considerable given the frequency of the data. Following this approach, we estimate these multipliers for a 1 year horizon. The results for these growth rates are presented in Figure 4.5.

[9]The regimes are classified using the smoothed probabilities displayed in Figure 4.4, using a threshold of 50% to determine the regime where the process is present.

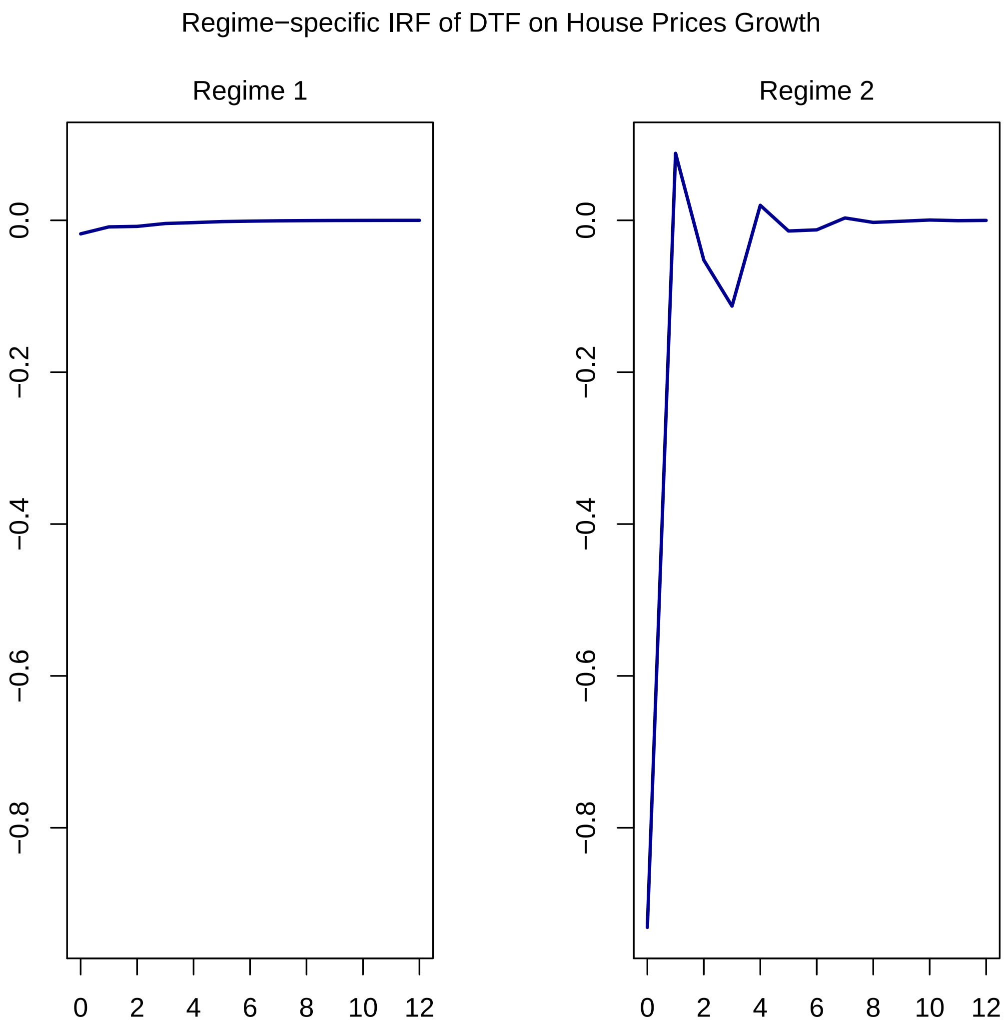


Figure 4.5: Regime-Specific Dynamic Multipliers of DTF on House Prices Growth.

These results suggests that under regular economic circumstances (regime 1), the effect is negative but modest and with a high persistence pace, meaning that the house prices growth should be slowed on the short run, but monetary policy has not very important implications whatsoever.

On the other hand, when the house prices growth rates are on periods of volatility, when the DTF increases by 100 basis points, the house prices growth rates dramatically decreased. Under these circumstances, the agents are more aware of the central bank's policy decisions and their reaction once there are changes in the policy rates is almost immediate. On the later periods, the house prices growth rates fluctuate for eight additional months before finally stabilizing in 0 at month ten. This implies strong short run implications of the central bank actions on the house prices, with a fast decaying effect in the long run.

5 Concluding Remarks

Several authors have been studying house price behavior in the last decade due to the economic downturn in 2008. Most of them find evidence of monetary policy effects on house price inflation as well as transmission mechanism in housing market to economic variables.

In this paper we have proposed a theoretical and empirical model for testing asymmetric effects of monetary policy on house prices. We use a user-cost approach to model the relationship between interest rates and house prices growth. We apply a Markov-switching model for the purpose of taking into account the regime dependent behavior of our variables. We also propose a dynamic multiplier regime dependent approach to model the response of house price inflation to an increase of one hundred basis points to DTF rates.

In the first place, we can conclude that house prices have two regimes: one that is less common than the other but with higher volatility than the regular state, and another which occurs more frequently and is characterized by higher persistence of the house prices growth rates. Our main finding is that an increase in monetary policy reduces house price inflation, specially on high volatility periods of house prices, where the effect is approximately of 93 basis points, while it has a negligible effect during regular periods. We can define the high volatility period as one in which housing activity stopped and mortgage lending decreased almost to zero (ANIF, 2011), facts that cause an irregular behavior in house prices which fluctuate around a stable mean.

Further research may be developed in order to look for possible transmission through house market channels to consumption, such as a balance sheet channel in Colombia taking into account different states of variables[10].

[10] A Markov-Switching VAR is a recommended tool for this purpose.

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Appendix A Life-Cycle Housing Model

A.1 Demand Side

Skaarup & Bodker (2010) provided a life-cycle model in which the representative household maximizes its discounted utility function choosing between two goods: a composite consumption good (c_t) and housing services (h_t), discounting it by the rate of time preference (θ). Lifetime utility can be written as:

$$max_{c,h}\ E_0 \sum_{t=0}^{\infty} \frac{1}{(1+\theta)^t} U(c_t, h_t) \tag{A.1}$$

The representative household maximizes the equation A.1 subject to the following budget constraint:

$$A_t = (1+r)A_{t-1} + y_t - c_t - p_t^h h_t \tag{A.2}$$

Where A_t is real financial wealth. r is the constant real riskless return on the equilibrium asset portfolio (includes housing equity and bonds), y_t is real income, and p_t^h is the real house price, while the price of c_t is numeriare. h_t is the real housing consumption, and it equals the user cost uc_t times the housing stock H_t. The present value of equation A.2 is:

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} c_t = A_0 + \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} (y_t - p_t^h h_t) \tag{A.3}$$

Then the maximization problem is solved via Lagrange function,

$$\mathscr{L}(c,h) = \frac{1}{(1+\theta)^t} U(c_t, h_t) + \lambda \left[A_0 + \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} (y_t - p_t^h h_t - c_t) \right]. \tag{A.4}$$

We can obtain the following first order conditions (FOC):

$$\begin{aligned}
\mathscr{L}(c_t, h_t)_{c_t} &= \frac{1}{(1+\theta)^t} U_c(c_t, h_t) - \frac{\lambda}{(1+r)^t} = 0 \\
\mathscr{L}(c_t, h_t)_{h_t} &= \frac{1}{(1+\theta)^t} U_h(c_t, h_t) - \frac{\lambda}{(1+r)^t} p_t^h = 0 \\
\mathscr{L}(c_{t+1}, h_{t+1})_{c_{t+1}} &= \frac{1}{(1+\theta)^{t+1}} U_c(c_{t+1}, h_{t+1}) - \frac{\lambda}{(1+r)^{t+1}} = 0 \\
\mathscr{L}(c_{t+1}, h_{t+1})_{h_{t+1}} &= \frac{1}{(1+\theta)^{t+1}} U_h(c_{t+1}, h_{t+1}) - \frac{\lambda}{(1+r)^{t+1}} p_{t+1}^h = 0.
\end{aligned} \tag{A.5}$$

The marginal rate of substitution (MRS) between composite consumption and housing services is equal to the relative prices,

$$\frac{U_h(c_t,h_t)}{U_c(c_t,h_t)} = p_t^h \tag{A.6}$$

Equation A.6 shows the intratemporal substitution. Standard Euler-equations for consumption and housing services are shown below expressing intertemporal substitution,

$$\frac{U_c(c_t,h_t)}{U_c(c_{t+1},h_{t+1})} = \beta(1+r) \tag{A.7}$$

$$\frac{U_h(c_t,h_t)}{U_h(c_{t+1},h_{t+1})} = \beta(1+r)(1+\pi_t^h) \tag{A.8}$$

Where π_t^h is the gross growth of housing prices and $\beta = \frac{1}{(1+\theta)}$. Following Skaarup & Bodker (2010) we assume a constant risk aversion utility function (CRRA),

$$U(c_t,h_t) = \frac{\gamma c_t^{(1-\phi)} + (1-\gamma)h_t^{(1-\phi)}}{1-\phi} \tag{A.9}$$

Using it in the marginal rate of substitution A.6 we find:

$$p_t^h = \frac{1-\gamma}{\gamma}\left(\frac{c_t}{h_t}\right)^{\phi} \tag{A.10}$$

Taking into account that $h_t = uc_t H_t$ and taking logarithms to equation A.10 we obtain

$$log(p_t^h) = \phi[log(c_t) - log(uc_t) - log(H_t)] + log\left(\frac{1-\gamma}{\gamma}\right) \tag{A.11}$$

We can also rewrite the Euler equations as,

$$c_{t+1} = c_t(\beta(1+r))^{\frac{t}{\phi}} \tag{A.12}$$

$$h_{t+1} = h_t(\beta(1+r)(1+\pi_t^h))^{\frac{t}{\phi}} \tag{A.13}$$

Now incorporating A.12 and A.13 in A.3, we have

$$c_t \sum_{\tau=t}^{\infty}(\beta(1+r))^{\frac{\tau}{\phi}}(1+r)^{-\tau} = A_t + \sum_{\tau=t}^{\infty}(1+r)^{-\tau}y_\tau - h_t \sum_{\tau=t}^{\infty}(\beta(1+r)(1+\pi_\tau))^{\frac{\tau}{\phi}}(1+r)^{-\tau}p_\tau^h \tag{A.14}$$

Then we can rewrite A.14 as

$$c_t = \mu(A_t + \widehat{y}_t - v_t p_t^h h_t), \tag{A.15}$$

where $\mu = \left(\sum_{\tau=t}^{\infty}(\beta(1+r))^{\frac{\tau}{\phi}}(1+r)^{-\tau}\right)^{-1}$ is the propensity to consume, the net present value of future income flows is $\widehat{y}_t = \sum_{\tau=t}^{\infty}(1+r)^{-\tau}y_\tau$, and $\nu_t p_t^h = \sum_{\tau=t}^{\infty}\beta^{\frac{\tau}{\phi}}(1+r)^{(\frac{1+\phi}{\phi})\tau}(1+\pi)^{\frac{1-\phi}{\phi}\tau}p_t^h$ and $p_t^h = p_\tau^h(1+\pi_\tau)^\tau$. Expressing A.15 in log terms using Taylor expansions,

$$log(c_t) = log(\mu) + \psi_A log(A_t) + \psi_Y log(\widehat{y}_t) - \psi_H(log(\nu_t) + log(p_t^h) + log(h_t))$$

where $\psi_A = \frac{A}{A+\widehat{y}-\nu p^h h}$,$\psi_Y = \frac{\widehat{y}}{A+\widehat{y}-\nu p^h h}$ and $\psi_H = \frac{\nu p^h h}{A+\widehat{y}-\nu p^h h}$. Variable names without time subscript means steady state.

Finally, plugging the last equation into the A.11 we obtain the house price equation

$$log(p_t^h) = \eta_A log(A_t) + \eta_Y log(\widehat{y}_t) - \eta_h[log(H_t) + log(uc_t)] + \chi_0, \quad \text{(A.16)}$$

where $\eta_A = \frac{\phi\psi_A}{1+\phi\psi_H}$,$\eta_Y = \frac{\phi\psi_Y}{1+\phi\psi_H}$, $\eta_H = \frac{\phi(1+\psi_H)}{1+\phi\psi_H}$ and $\chi_0 = \frac{\phi}{1+\phi\psi_H}log(\mu) - \frac{\phi\psi_H}{1+\phi\psi_h}log(\nu) + \frac{1}{1+\phi\psi_H}log\left(\frac{1-\gamma}{\gamma}\right)$. This equation shows that house prices $\left(p_t^h\right)$ depends negatively on user costs (uc_t) and housing stock (H_t), and directly on discounted income flows $(\widehat{y}_t)$.

Meen (2001) describes the user cost as

$$uc_t = p_t^h\left[i_t - \pi_t - \frac{\Delta p_t^{h,e}}{p_t^h}\right], \quad \text{(A.17)}$$

where i_t is the market interest rate, π_t is general inflation rate and the last term is the expected real capital gain. Plugging A.17 in equation A.16 we obtain,

$$log(p_t^h) = \rho_A log(A_t) + \rho_Y log(\widehat{y}_t) - \rho_H log(H_t) - \rho_{uc} log\left(i_t - \pi_t - \frac{\Delta p_t^{h,e}}{p_t^h}\right) + \zeta_0,$$

where $\rho_A = \eta_A/(1+\eta_H)$,$\rho_Y = \eta_Y/(1+\eta_H)$, $\rho_{uc} = \eta_H/(1+\eta_H)$ and $\zeta_0 = \chi_0/(1+\eta_H)$. Again using Taylor expansions for the logarithm of user cost we obtain,

$$log(p_t^h) = \rho_A log(A_t) + \rho_Y log(\widehat{y}_t) - \rho_H log(H_t) - \rho_i i_t + \rho_\pi \pi_t - \rho_\Delta \frac{\Delta p_t^{h,e}}{p_t^h} + \zeta_0, \quad \text{(A.18)}$$

where $\rho_i = \rho_{uc} i^*/(i^* - \pi^* + \frac{\Delta p^*}{p^*})$, $\rho_\pi = \rho_{uc}\pi^*/(i^* - \pi^* + \frac{\Delta p^*}{p^*})$ and $\rho_\Delta = \rho_{uc}\frac{\Delta p^*}{p^*}/(i^* - \pi^* + \frac{\Delta p^*}{p^*})$. Where star variables (x^*) means steady state value of each variable.

A.2 Supply Side

Using an intertemporal capital stock approach we model the supply side as Skaarup and Bødker did based on Poterba (1984) and Madsen et al. (2011). Firm's profits are given by the value of housing sales $p_t^h H_t$, minus the cost of investment in housing by firms, that is divided in direct costs $C_t^I I_t$ and some convex costs of investment $C(I_t, H_t) = \frac{1}{2}\beta \frac{I_t^2}{H_t}$. The representative firm chooses the optimal amount of investment by maximizing the net present value of profits,

$$max_{H,I}\ \Pi = \sum_{t=0}^{\infty}(1+r)^{-t}[p_t^h H_t - C_t^I I_t - C(I_t, H_t)] \tag{A.19}$$

Subject to the evolution of housing stock,

$$H_{t+1} = (1+\delta)H_t + I_t \tag{A.20}$$

Then, the Lagrangian is

$$\mathscr{L} = \sum_{t=0}^{\infty}(1+r)^{-t}[p_t^h H_t - C_t^I I_t - C(I_t, H_t)] + \sum_{t=0}^{\infty}\lambda[(1-\delta)H_t + I_t - H_{t+1}] \tag{A.21}$$

Defining the shadow price of houses that is equal to the market value of houses relative to the replacement value, $q_t = (1+r)^t \lambda$, we rewrite A.21 as,

$$\mathscr{L} = \sum_{t=0}^{\infty}(1+r)^{-t}[[p_t^h H_t - C_t^I I_t - C(I,H)] + q_t[(1-\delta)H_t + I_t - H_{t+1}]] \tag{A.22}$$

Solving the problem yields the first order conditions of housing stock and investment in housing,

$$\begin{aligned} \mathscr{L}_I &= (1+r)^{-t}(-C_t^I - C_I(I,H) + q_t) = 0 \\ \mathscr{L}_H &= (1+r)^{-t}(-p_t^H - C_H(I,H) + q_t(1-\delta)) - (1+r)^{-(t-1)}q_{t-1} = 0 \end{aligned} \tag{A.23}$$

Rearranging them we find,

$$\begin{aligned} C_I(I,H) &= C_t^I - q_t \\ p_t^H &= C_H(I_t, H_t) - q_t(1-\delta)) + (1+r)q_{t-1} = 0 \\ lim(1+r)^{-t}q^t H^t &= 0 \end{aligned} \tag{A.24}$$

Equation A.24 shows that firms invest to the point where the price of houses is equal to investment costs. Then the net present value of house prices is,

$$q_t = \frac{1}{1-\delta}\sum_{t=0}^{\infty}\left(\frac{(1-\delta)(1+\pi_t)}{(1+r)}\right)^t p_t^H - \frac{1}{1-\delta}\sum_{t=0}^{\infty}\left(\frac{(1-\delta)}{(1+r)}\right)^t C_H(H_t, I_t) \tag{A.25}$$

Plugging A.25 on the first equation of A.24, rearranging it and log-linearizing around one, we obtain the firm's optimal investment at time zero (recall $C_H(I_t, H_t) = -\frac{1}{2}\beta\left(\frac{I}{H}\right)$),

$$\beta\left(\frac{I_t}{H_t}\right) - \frac{\sigma}{2(1-\delta)}\beta\left(\frac{I_t}{H_t}\right)^2 = \frac{1}{1-\delta}\sum_{t=0}^{\infty}\left(\frac{(1-\delta)(1+\pi)}{1+r}\right)p_t^H - C_t^I = \left(\frac{\mu}{1-\delta}\right)p_t^H - C_t^I,$$

where $\mu = \left(1-\frac{(1-\delta)(1+\pi)}{(1+r)}\right)^{-1}$ and $\sigma = \left(1-\frac{(1-\delta)}{(1+r)}\right)^{-1}$. Log-linearizing left side around one we obtain:

$$\begin{aligned}
log\left[\left(\frac{I_t}{H_t}\right) - \frac{\sigma}{2(1-\delta)}\beta\left(\frac{I_t}{H_t}\right)^2\right] &= log\left[\beta\left(\frac{I_t}{H_t}\right)\left(1-\frac{\sigma}{2(1-\delta)}\frac{I_t}{H_t}\right)\right] \\
&= log(\beta) + log\left(\frac{I_t}{H_t}\right) + log\left(1-\frac{\sigma}{2(1-\delta)}\frac{I_t}{H_t}\right) \\
&\approx log(\beta) + \left(\frac{I_t}{H_t}\right) - 1 + loglog\left(1-\frac{\sigma}{2(1-\delta)}\right) + \left(1-\frac{1}{\frac{2(1-\delta)}{\sigma}-1}\right)log\left(\frac{I_t}{H_t}\right) \\
&= log(\beta) + log\left(1-\frac{\sigma}{2(1-\delta)}\right) + \left(1-\frac{1}{\frac{2(1-\delta)}{\sigma}-1}\right)log\left(\frac{I_t}{H_t}\right).
\end{aligned}$$

We also log-linearize around one the right hand side resulting in,

$$log\left(\left(\frac{\mu}{1-\delta}\right)p_t^H - C_t^I\right) \approx log\left(\frac{\mu}{1-\delta}-1\right) + \frac{1}{1-\left(\frac{1-\delta}{\mu}\right)}log(p_t^H) - \left(\frac{1}{\frac{\mu}{1-\delta}-1}\right)log(C_t^I).$$

Using both side equation and isolating $\left(\frac{I_t}{H_t}\right)$ we find,

$$log\left(\frac{I_t}{H_t}\right) = \frac{1}{\phi}\left[\rho + \frac{\omega}{\omega-1}log(p_t^H) - \frac{1}{\omega-1}log(C_t^I)\right], \tag{A.26}$$

Where $\rho = \left(log\left(\frac{\mu}{1-\delta}-1\right)\right) - log(\beta) - log\left(1-\frac{\sigma}{2(1-\delta)}\right)$, $\omega = \frac{\mu}{1-\delta} > 1$, and $\phi = 1 - \frac{1}{\frac{2(1-\delta)}{\sigma}-1} > 0$. This states that housing investment depends negatively on investment cost and positively in housing price. In addition, we can also see that house prices depends positively on investment amount and its costs.

A.3 Central Bank

The way how monetary policy influences the house price is through user cost channel. Where the central bank increases their interest rates (a contractionary policy) market interest rates increase. The monetary policy followed by the central bank can be written as,

$$i_t = \rho_s i_{t-1} + (1-\rho_s)[\rho_i \bar{i} + \rho_\pi(\pi_t - \bar{\pi})] + \varepsilon_t. \tag{A.27}$$

Equation A.27 is a version of the FISCO model monetary policy equation for Colombia (Rincon et al., 2014). This shows that the central bank responds to deviation in inflation from target $\bar{\pi}$. It also is adjusted in a proportion ρ_s to lagged value.

Regime Shifts In Monetary Policy

Rabanal (2004) describes a Markov Switching Taylor rule which can be implemented in our model, thus rewriting equation A.27 as,

$$i_t = \rho_s(s_t) i_{t-1} + (1-\rho_s(s_t))[\rho_i(s_t) \bar{i} + \rho_\pi(s_t)(\pi_t - \bar{\pi})] + \varepsilon_t. \tag{A.28}$$

A.4 Solving the Model

In order to find the equilibrium housing price equation, we take up equation (A.18), (A.26) and (A.28),

$$log(p_t^h) = \rho_A log(A_t) + \rho_Y log(\widehat{y}_t) - \rho_H log(H_t) - \rho_i i_t + \rho_\pi \pi_t - \rho_\Delta \frac{\Delta p_t^{h,e}}{p_t^h} + \zeta_0 \tag{A.29}$$

$$log\left(\frac{I_t}{H_t}\right) = \frac{1}{\phi}\left[\rho + \frac{\omega}{\omega - 1} log(p_t^H) - \frac{1}{\omega - 1} log(C_t^I)\right] \tag{A.30}$$

$$i_t = \rho_s(s_t) i_{t-1} + (1-\rho_s(s_t))[\rho_i(s_t) \bar{i} + \rho_\pi(s_t)(\pi_t - \bar{\pi})] + \varepsilon_t \tag{A.31}$$

Thereafter we are going to write the expected capital gains $\frac{\Delta p_t^{h,e}}{p_t^h}$ as CG_t. Isolating $log(H_t)$ from equation (A.30) we obtain,

$$log(H_t) = log(I_t) + \frac{1}{\phi(\omega - 1)} log(C_t^I) - \frac{\rho}{\phi} - \frac{\omega}{\phi(\omega - 1)} log(p_t^H). \tag{A.32}$$

Inserting (A.31) in (A.29) we have,

$$log(p_t^h) = \widetilde{\alpha_0(S_t)} - \widetilde{\alpha_i(S_t)} i_{t-1} + \widetilde{\alpha_\pi(S_t)} \pi_t - \widetilde{\alpha_\Delta} CG_t + \widetilde{\alpha_A} log(A_t) + \widetilde{\alpha_y} log(\hat{y}_t) - \widetilde{\alpha_H} log(H_t), \quad \text{(A.33)}$$

where,

$$\widetilde{\alpha_0(S_t)} = [\rho_s(S_t) - 1][\rho_i \rho_i(S_t)\bar{i} - \rho_i \rho_\pi(S_t)\bar{\pi}] + \zeta_0$$
$$\widetilde{\alpha_i(S_t)} = \rho_i \rho_s(S_t)$$
$$\widetilde{\alpha_\pi(S_t)} = [\rho_i \rho_\pi(S_t)[\rho_s(S_t) - 1] + \rho_\pi]$$
$$\widetilde{\alpha_\Delta} = \rho_\Delta, \widetilde{\alpha_A} = \rho_A, \widetilde{\alpha_y} = \rho_y, \widetilde{\alpha_H} = \rho_H.$$

Now we replace $log(H_t)$ by (A.32), and replacing $\chi_{-H} = \widetilde{\alpha_0(S_t)} - \widetilde{\alpha_i(S_t)} i_{t-1} + \widetilde{\alpha_\pi(S_t)} \pi_t - \widetilde{\alpha_\Delta} CG_t + \widetilde{\alpha_A} log(A_t) + \widetilde{\alpha_y} log(\hat{y}_t)$ for simplicity by now, we find,

$$log(p_t^h) = \chi_{-H} - \rho_H log(I_t) - \frac{\rho_H}{\phi(\omega - 1)} log(C_t^I) + \frac{\rho_H \rho}{\rho} + \frac{\rho_H \omega}{\phi(\omega - 1)} log(p_t^H)$$

Taking into account that in equilibrium $p_t^h = p_t^H$, replacing again χ_{-H} and rearranging the model to isolate p_t^h we obtain the equilibrium housing price,

$$\begin{aligned} log(p_t^h) = \beta_0(S_t) - \beta_i(S_t) i_{t-1} + \beta_\pi(S_t)\pi_t + \beta_A log(A_t) + \\ \beta_y log(\hat{y}_t) - \beta_I log(I_t) - \beta_C log(C_t^I) - \beta_\Delta CG_t \end{aligned} \quad \text{(A.34)}$$

where,

$$\beta_0(S_t) = \frac{\widetilde{\alpha_0(S_t)} + \rho_0}{1 - \frac{\widetilde{\alpha_H}\omega}{\phi(\omega-1)}}, \beta_i(S_t) = \frac{\widetilde{\alpha_i(S_t)}}{1 - \frac{\widetilde{\alpha_H}\omega}{\phi(\omega-1)}}, \beta_\pi(S_t) = \frac{\widetilde{\alpha_\pi(S_t)}}{1 - \frac{\widetilde{\alpha_H}\omega}{\phi(\omega-1)}} \beta_A = \frac{\widetilde{\alpha_A}}{1 - \frac{\widetilde{\alpha_H}\omega}{\phi(\omega-1)}}$$
$$\beta_y = \frac{\widetilde{\alpha_y}}{1 - \frac{\widetilde{\alpha_H}\omega}{\phi(\omega-1)}} \beta_I = \frac{\widetilde{\alpha_H}}{1 - \frac{\widetilde{\alpha_H}\omega}{\phi(\omega-1)}} \beta_C = \frac{\rho_C}{1 - \frac{\widetilde{\alpha_H}\omega}{\phi(\omega-1)}} \beta_\Delta = \frac{\widetilde{\alpha_\Delta}}{1 - \frac{\widetilde{\alpha_H}\omega}{\phi(\omega-1)}}.$$

Appendix B   Markov Switching Unit Root Test

Villa et al. (2014) described the Markov Switching Unit Root (MSUR) procedure for Colombian inflation case. Based on their work, in general we can write the auxiliary regression of the Augmented Dickey Fuller test (ADF), as shown in equation B.1. This model takes into account that coefficients and variances are state dependent, that means parameters depends on a stochastic and unobserved state variable $s_t \in \{0,1\}$,

$$\Delta y_t = C_{s_t} + \rho_{s_t} y_{t-1} + \theta_{s_t} + \sum_{j=1}^{K} \eta_{j,S_t} \Delta y_{t-j} + \nu_j, \tag{B.1}$$

s_t follows a Markov order process. Then its transition probabilities are defined by the following expressions: $P\{s_t = 1|s_{t-1} = 1\} = p$, $\{s_t = 2|s_{t-1} = 2\} = q$, $P\{s_t = 2|s_{t-1} = 1\} = 1-p$ and $P\{s_t = 1|s_{t-1} = 2\} = 1-q$. As the regular ADF test, the MSUR is based on the *t-statistic* corresponding to the coefficient ρ_{s_t}. The null hypothesis assumes the existence of unit root, that is $\rho_{s_t} = 0$, the alternative hypothesis means that y_t follows a stationary process $\rho_{s_t} < 0$. The Markov Switching Augmented Dickey Fuller test (MS-ADF) follows these steps:

1. Equation B.1 should be estimated under the null hypothesis and its residuals must be grouped into two subsets, A and B, where $A \cap B = \emptyset$:

   (a) Subset A contains residuals from regime 1[11].

   (b) Subset B contains residuals from regime 2.

2. We construct a number of Q subvectors of A^q and B^q, with $q = 1,...,Q$, under sampling with repetition of subsets A and B. Respecting their position using filtered probabilities, we construct Q vectors $V^q = A^q \cup B^q$, for $q = 1,...,Q$.

3. Generate a dichotomous state variable s_t^* taking into account the filtered probabilities.

4. Now, we generate Q realizations of variable y^q, $q = 1,...,Q$ using disturbances computed in step two, for all $q = 1,...,Q$,

   $$\Delta y_t^q = C_1(1-s_t^*) + C_2 s_t^* + \theta_1(1-s_t^*) + \theta_2 s_t^* + \sum_{j=1}^{K} [\eta_{j,1}(1-s_t^*)\Delta y_{t-j} + \eta_{j,2} s_t^* \Delta y_{t-j}] + \varepsilon_t^q. \tag{B.2}$$

   Equation B.2 is the same equation B.1 but under null hypothesis($\rho_{s_t} = 0$).

5. We estimate equation B.1 for each of the Q vectors and *t-statistics* are stored in a vector of

[11] Residuals are assigned to their respective subset based on filtered probabilities.

size $Q \times 1^{12}$.

6. Critical values of t-statistics are computed for 1%,5% and 10% levels of significance, to compare them with the one estimated in the equation B.1 for the original data.

Appendix C Data Information

We use different sources for variable construction as is shown in Table C4, the dataset is on a monthly frequency ranging from January 1994 to December 2015:

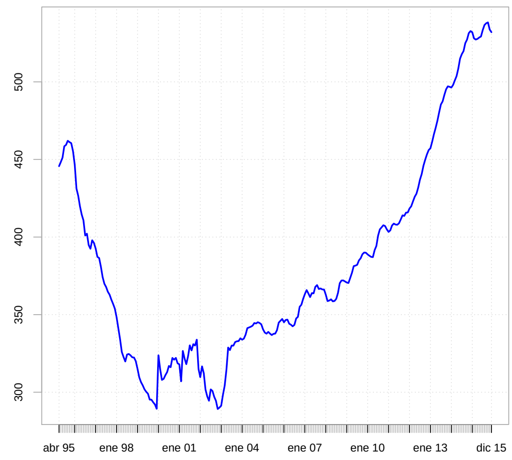
Variable	**Variable modeled**	**Source**
New housing price index	House prices	DNP-Colombia
Interest Rate of 90-day certificate of deposit (DTF)	Monetary policy rate	Banco de la República de Colombia
Inflation rate CPI index	Inflation rate	DANE-Colombia
Monthly Gross Domestic Product	Income and households wealth	Banco de la República de Colombia- Authors calculations
Housing construction licenses (approved squared meters)	Housing supply	CAMACOL
Housing cost construction index	Cost of building houses for firms	DANE-Colombia

Table C4: Data sources.

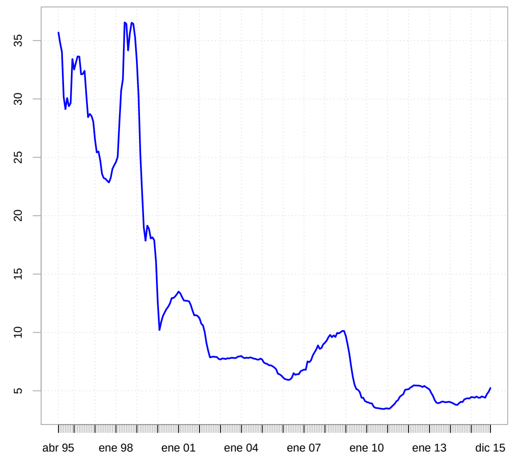
Our housing index variable is calculated using Bogotá and Chía information, unfortunately, information on a national level is unavailable at the moment due to insufficient information for this index to be calculated. We compute logarithms values for all variables except interest and inflation rates. In addition, use the Hodrick-Prescott filter (Hodrick & Prescott, 1997) to obtain the tendency of housing construction licenses given its irregular behavior. We use quarterly constant prices GDP and using the methodology proposed by Silva & Cardoso (2001) we compute the monthly observations based on IPI index (Industrial Production Index).

We present some graphs of these data below: the series are plotted both in levels and first-difference or growth rates when relevant.

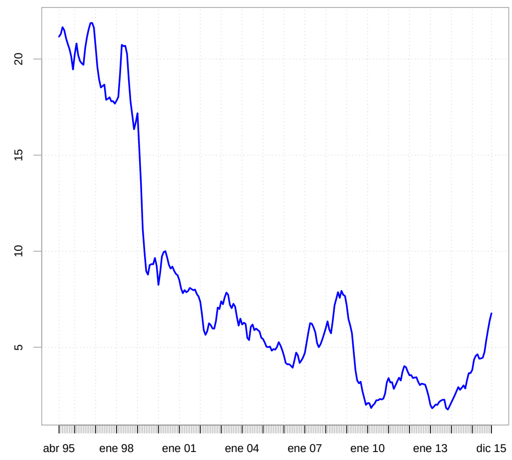
[12]"*T-statistics are constructed with the standard deviations from the negative of the inverse of the Hessian matrix associated with the optimization procedure that maximizes the likelihood function.*" Villa et al. (2014).



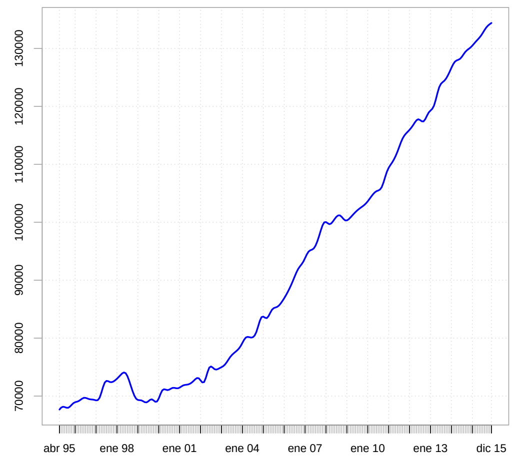
(a) Real house price index



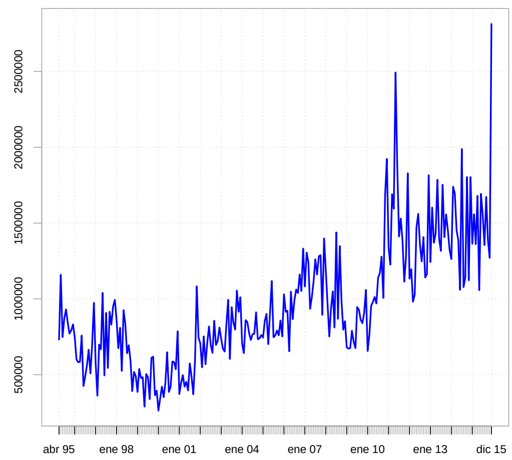
(b) DTF rate



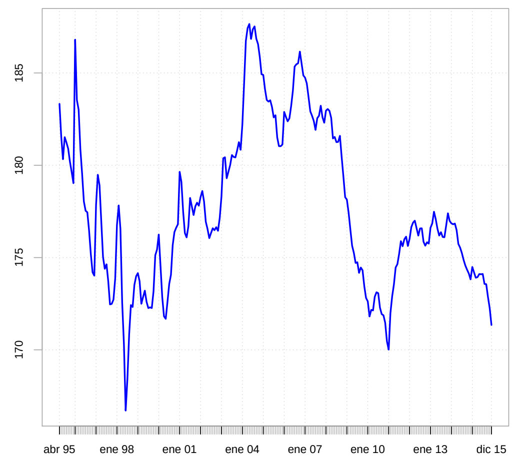
(c) Inflation rate



(d) Real GDP

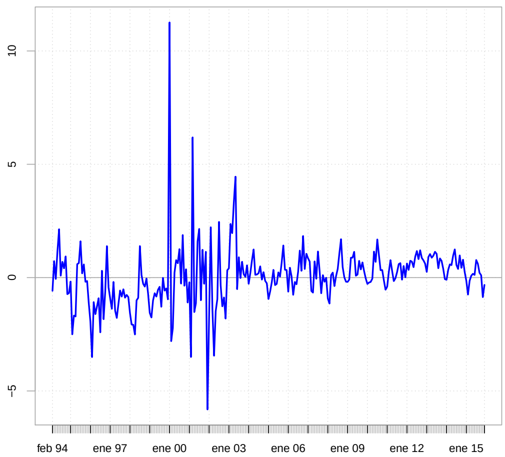


(e) Licences approved

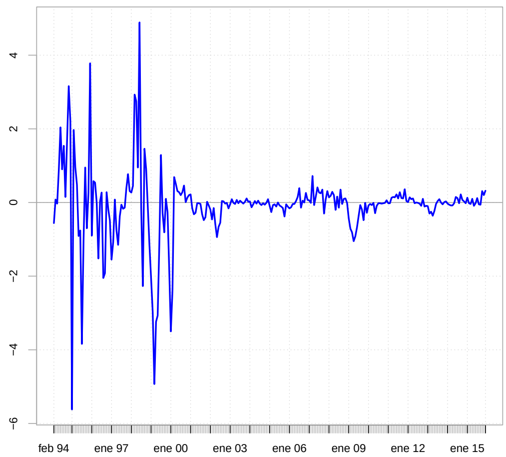


(f) Housing construction costs

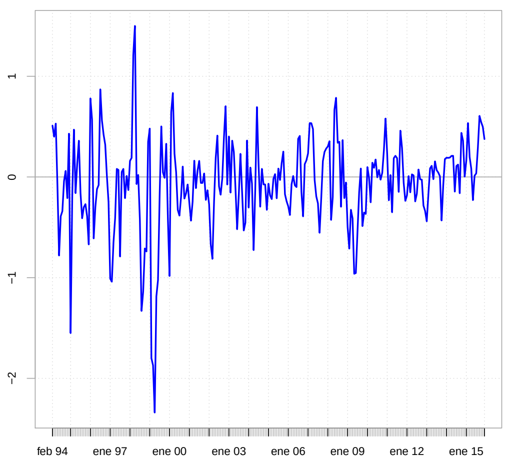
Figure C1: Variables in original levels.



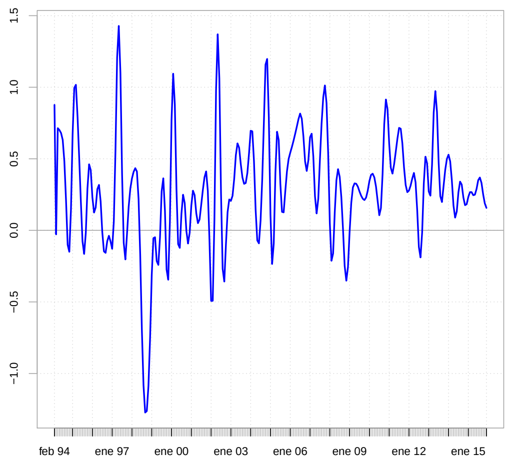
(a) House price growth



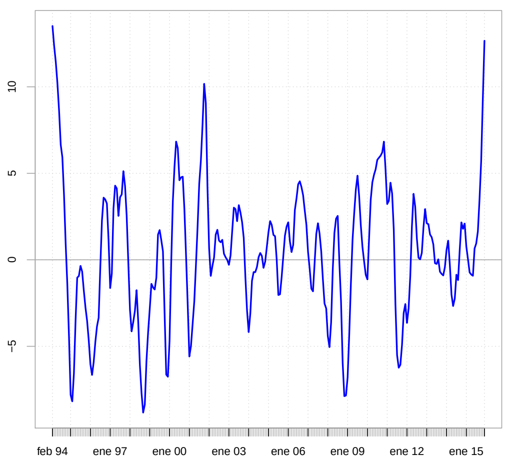
(b) DTF difference



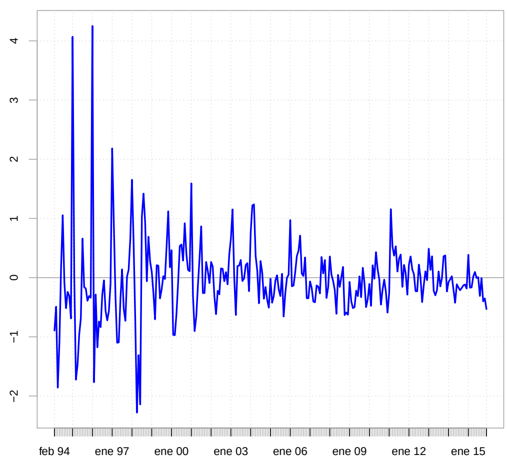
(c) Inflation difference



(d) GDP Growth



(e) Licences tendency growth



(f) Construction costs growth

Figure C2: Variables in growth and differences.

Appendix D Empirical Model Output

In this Appendix we present the full information on the model estimated, including the parameter standard errors and the t-statistics.

Regime-Switching Parameters

Regime	**1**				**2**			
	Parameter	Std	T-Stat	P-Value	Parameter	Std	T-Stat	P-Value
c_{s_t}	0.0177	0.0484	0.3657	0.7146	-0.2754	0.3337	-0.8253	0.4092
$\widehat{\gamma_{\Delta i,s_t}}$	-0.0178	0.0503	-0.3539	0.7234	-0.9312	0.4456	-2.0898	0.0366
$\widehat{\gamma_{\Delta \pi,s_t}}$	-0.0545	0.1237	-0.4406	0.6595	-2.1608	0.8926	-2.4208	0.0155
$\widehat{\phi_{1,s_t}}$	0.4850	0.0709	6.8406	0.0000	-0.0948	0.1253	-0.7566	0.4493
$\widehat{\phi_{2,s_t}}$	0.2132	0.0721	2.9570	0.0031	0.0472	0.1305	0.3617	0.7176
$\widehat{\phi_{3,s_t}}$	-0.0850	0.0645	-1.3178	0.1876	0.1312	0.1315	0.9977	0.3184

No Regime-Switching Parameters

	Parameter	Std	T-Stat	P-Value
$\widehat{\gamma_Y}$	-0.0001	0.0033	-0.0303	0.9758
$\widehat{\gamma_L}$	0.0464	0.0129	3.5969	0.0003
$\widehat{\gamma_C}$	0.1595	0.0935	1.7059	0.0880

Table D5: Model estimation of equation 2.17