Capstone Final Project

A quantitative approach for product portfolio optimization

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Abstract

Nowadays product proliferation is a very common issue for companies, uncontrolled product launches affect revenue, profit and service level, consequently there is a need to reduce the portfolio. In this project, we propose an optimization method for portfolio rationalization based on substitutability and average revenue. In order to transform the substitutability into a quantitative criterion, a Markov chain approach was implemented. This approach describes the substitution behavior and allows to calculate the redistribution of customers in the remaining SKUs. For each possible portfolio, there is a Markov chain that must be evaluated to know the future revenue performance. So, the number of possible solutions and the complexity of the problem increase exponentially as the number of SKUs increases. A Tabu search metaheuristic was proposed to solve this combinatorial problem.

Since all the companies do not have the same needs, requirements and expectations about the portfolio rationalization, two different contexts were defined. First context refers to companies that have no data input for the model because they have not done analysis about the rationalization. While second context refers to companies that have already defined a constraint for the reduction, the minimum percentage of SKUs to remove or the maximum revenue that the company is willing to lose. Aiming to evaluate the performance of the designed model, we simulated a case of study where a company is trying to reduce a portfolio of sixty products. Finally, from the analysis of the results we provided some insights about the way the model selects the products according to their revenue, preference and substitutability levels.

Key words: Optimization method, SKU rationalization, substitutability, average revenue, Markov chain, Tabu search

1. Problem statement and Justification

1.1. Problem statement

Along this project, the acronym SKU will be used to refer to each item from the company product portfolio. "*The stock keeping units or SKUs refer to items of stock that are completely specific as to function, style, size, color, and, usually location*" (Silver et al., 1998, p. 32; van Kampen, Akkerman, & Van Dock 2012). Nowadays, multiple variants of the same product coexist and each of these variants make a new SKU for the company (Capegmini 2007). Therefore, SKUs are often used to identify the production and logistics characteristics of each product variation such as the color, size, price, production levels, inventory rotation, packaging, distribution channel, market niche, among others. For instance, a specific product can be the "mother" of many "children products" that are different between them in at least one characteristic; this is how SKUs proliferate and hence the product portfolio. The main cause for such proliferation is that companies try to supply the necessities of customers, and to do so, they proliferate SKUs to adapt their products. This customer focused innovation has produced more than 500 shampoos at Wal-Mart, 85 30-inch televisions at Circuit City, 80 varieties of pens at Office Depot and 15 versions of a single brand of toothpaste at the local drugstore (Byrne 2007).

Although it seems that SKU proliferation is a result of innovation and the company's commitment to fulfil customer needs, which are positive arguments, there are also some negative consequences. By proliferating SKUs, shopping becomes stressful and sometimes the customer has to sort through a half dozen varieties to find a suitable substitute, because the product the customer needs is "out of stock". The out-of-stock products appear when retailers do not provide enough space to the traditional but popular brands because new SKUs are added. Therefore, SKU proliferation could reduce the customer satisfaction. Similarly, when there is a huge variety of products, forecasting and planning become difficult, and more of everything is required: more supervisors, training, inventory, production-line changeovers, capital, and time and expense in product development. Those resources increase the product cost paid by the consumer (Byrne 2007). Van Kampen, Akkerman, & Van Dock (2012) concluded that companies that sell wide variety of SKUs often have troubles with controlling production and inventory systems. Additionally, Gilliland (2011) conclude each SKU represents a cost for the company; you might have to occupy storage, perhaps periodically require production and distribution capacity. In case of low-volume sales SKUs, this cost generates a loss of revenues in the company. Some of the offering may not be worth the effort needed to provide them. Taking into account what was said before, the reduction or rationalization of SKUs could be a way to reduce the costs and increase the profit margins of the company.

SKU rationalization is the process of evaluating each item based on its contribution to overall business objectives. Those SKUs that fail to meet the company's requirements (in terms of revenue or other financial or strategic metrics) should then be eliminated (Gilliland 2011). Nevertheless, SKU rationalization is not that simple. One key factor that explains why companies tend to keep many SKUs is substitutability. The reason SKU rationalization struggles so often is that there are items on the shelf, which from the consumer's standpoint, are unique (Fields 2006). Sometimes, retailers do not rationalize their SKUs because they fear the potential consequences. For instance, if the decision to remove a product is made solely on that product's performance, they may be losing a product that helps drive the sales of associated products. Worse, there is a risk of losing a key customer and never regaining their business (Quantum retail technology 2010). On the other hand, while pruning items is likely to save on costs, it may also seem likely to lose a small amount of revenue by no longer selling these products, for that reason some companies still refrain to SKU rationalization and prefer to keep the current portfolio. Moreover, the SKU rationalization is not appropriate when there are products newly released and are still establishing themselves in the marketplace, or it exists a contract to continue providing certain number of products to a retailer (Gilliland 2011).

1.2. Justification

The rationalization of SKUs can be done for several highly opportunistic situations, for example, making room for innovation, correcting shelf-space inequities and correcting out-of-stock issues (Fields 2006). Nowadays, there are methodologies and strategies that are used to rationalize SKUs and are mostly based on qualitative methods, which usually creates disputes between the points of view, not only about which methods should be used, but also on the implementation of those methods and their actual outcomes. In addition, the SKU rationalization implies an agreement of all the company's departments, and sometimes the elimination of SKUs may be beneficial for some departments (i.e., production or logistics) and counter-productive for others (i.e., marketing). Some strategies, such as the one reported by Bob Byrne (2007), consider a *customer first approach*, while Rahul Mittal, Navneet Sharma, Tarun Batran and Udit Maheshwari (2012) proposed a technique based on benefit measurement that the rationalization of SKUs provides.

Undoubtedly, there is a necessity for quantitative methods for SKU optimization that consider different aspects of the SKU's (remembrance, sales, gross margin, costs, and substitutability). As far as the authors know and based on the literature review presented in the next section, none of the methodologies reviewed consider several perspectives, coming from various areas of the company. Usually, very few indicators are taken into consideration, and there is a lack for a holistic view for making the right decision. Sometimes, authors do have several criteria in consideration; nevertheless, these criteria are only qualitative or quantitative. The methodologies proposed so far are then usually based by certain information, i.e., financial, marketing, production, etc. Therefore, the following research question arises *How to design a method based on quantitative and qualitative criteria to support the SKU rationalization process inside a company?* Our hypothesis is that a method based on quantitative and qualitative criteria, based on optimization techniques, allows having SKU assortments that are easier and more efficient to manage, and that are more profitable. By finding the right number of SKUs, companies can focus on those products that are important and are adding value (Quantum retail technology 2010).

2. Literature review

According to David A. Fields (2006), there is no single best practice for SKU rationalization, but there are various best practices that "proliferate their portfolios with marginally valuable line extensions". The author explains a series of steps in which a manufacturer can achieve such best practices. Step 1 focuses on identifying the consumer's purchase decision hierarchy. Step 2 deals with identifying substitutability. Step 3 must determine the optimal number of SKUs (diminishing returns), while step 4 determines sales and profit productivity. Last, step 5 finalizes with SKU selection. At the end of this article, the author emphasizes on the importance of SKU rationalization, stating "*As long as manufacturers proliferate their portfolios and as long as retailers build larger and larger stores, the need for SKU rationalization will continue to increase. The challenge is to use a SKU rationalization process which marries the strategic needs of manufacturers and retailers with rigorous data analysis in order to provide the best scenario for all involved—including consumers*".

Dave DeWalt (2005) declares that even if everyone is aware of the problems caused by having too many products, the benefits of dropping some are not clear on the Profit & Loss Statement (but the lost volume certainly is). Product line optimization (that is SKU optimization) requires a long-term perspective, unless there are capacity constraints. On the other hand, SKU optimization makes it hard to see the financial case, but it creates long-term benefits, such as reducing inventory-carrying costs, reducing changeover/short run costs, eliminating waste of scarce resources, sharpening employees focus on the star products, open slots for star products, support annual planning efforts. To achieve such product line optimization, the author proposes to follow these four steps: Step 1: identify candidates according to their volume sales and gross profit; Step 2: evaluate and develop action plans; Step 3: project the P&L impact of potential action plans; and Step 4: decide, communicate, and implement.

Capegmini (2007), in the white paper "The Business Case for Product Rationalization", suggests the creation of a measurement that involves level sales, profit margins and inventory in order to make an accurate analysis to rationalize SKUs. This measurement is the gross margin return of investment (GMROI), which means that for every dollar invested in inventory, there must be an amount of contribution made to the overall profitability. After the calculation of the GMROI, it is necessary to identify the category in which the product is. The categories are: consumer staples, family jewels, rationalization opportunity or niche performers. The next step in this methodology is to make a strategy based on the categorization, then differentiate products with high potential of those with low potential, and finally rationalize. The most important characteristic of this method is the holistic view that offers the GMROI.

In the article "Finally, a strategic way to cut unnecessary SKUs", Bob Byrne (2007) presents a strategic approach to SKU rationalization designed to eliminate high-volume but unnecessary SKUs. The strategy applies the Consumer First Approach. It is to use research on consumer preferences and switching behavior to design an optimal product portfolio based on what the consumer really wants. According to the author, the strategy that attacks the lowest volume SKUs is useless because reducing the products on the tail generate the minimal cost reduction impact and not make a significant change. Meanwhile eliminating redundant and unnecessary SKUs allows enhancing the brand image and increases growth opportunities because there is less shelf clutter, fewer out-of-stocks and more time to focus on true innovation.

Michael Gilliland (2011) argues that effective management of the product portfolio requires periodic pruning of extremely low volume products. In order to make the pruning, the author creates a Pareto chart ranking the revenue (or volume) of all the product offerings. Before eliminating the products of the bottom, the author suggests analyzing some aspects such as substitute products, inventory levels and order-fill rates, among others. Gilliland recommends that the process should be done periodically, at least annually, with the list of pruning candidates and the background information distributed throughout the organization. Appeals can be made, and some of these low-volume products can then be saved. However, the default decision is to prune unless there is a good justification for keeping the item. All the costs in management and planning time saved must be focusing on the more important items.

According to Mittal, Sharma, Batra and Maheshwari (2012), SKU rationalization follows a four-step life cycle, in the last step the benefit of removing those SKUs can be measured. The approach measures the benefit in terms of inventory, money and the benefit of the keeping SKUs. The four steps are the following: inception, that consists in identify non-performing SKU's based on several parameters such as the annual sales, gross margins, space productivity, etc. Inception also includes the creation of a business case in which you define the available solutions, their associated costs and benefits and which is the recommended solution. The second step is planning a detailed plan should be developed for the implementation, the success of the initiative lays on how robust the benefit measurement approach is. Next step is execution, is necessary to take in consideration the buy – in form stakeholders, nevertheless, one SKU rationalization does not solve the proliferation of SKU's problem, a program to continue monitoring the performance and metrics of the remain SKU's is the best option, because buyers and suppliers tend to "*introduce new SKU's from time to time"*. The last step is realization, as it was said before, the key to success in the SKU rationalization initiative is the measurement of the benefit of doing so, this measure should be easy to quantify and understand. There are three different mechanisms used to benefit measurement they are: inventory depletion measurement for rationalized SKUs only, in which the Inventory depletion of the rationalized SKUs is tracked. This approach provides an understanding of the impact on the inventory of the rationalized SKUs and helps to develop an exit strategy for the rationalized SKUS. The second approach is the holistic view of SKU rationalization impact at the organization level, it gives a global view of the inventory levels at the organization level, not only of the SKUs that were rationalized. Finally, the Holistic view assessment based on the review of inventory turns improvement from the SKU Rationalization programme this approach gives a holistic view of the overall impact in the inventory turns for the organization, in this measurement is necessary to define a baseline and the new measurements will be compared with the baseline. For these authors the annual sales in units and dollars, inventory turnover ratio and sales to inventory ratio are the principal criteria to consider.

Barry Berman (2011) describes how a firm can limit product proliferation without incurring reduced sales or lowering consumer loyalty. The author proposed a SKU proliferation reduction program based on several principles: resisting the temptation of asking consumers if a greater assortment is required; classifying goods into consumer behavior-based tiers; using inter-functional product pruning teams; practicing mass customization, placing absolute limits on product choice; and implementing effective strategies for product pruning. In the article, Berman considers a case study of successful product proliferation reduction, Clorox. Four years after the program implementation, over 90% of Clorox's SKUs had met sales and profit targets, and net sales per SKU had almost doubled.

On his thesis, submitted to the MIT Sloan School of Management, David Hilliard (2012) develops a process for SKU rationalization to reduce SKU complexity while maintaining sales volumes. In this process, it is recommended that SKUs with low volumes and high demand variation be targeted for rationalization. Hilliard implemented operational models to compute complexity costs associated with SKU complexity. The complexity cost is defined as the impact that SKU proliferation has on cash flows, inventory, and avoidable costs. The author computes the impact on cash flows by modeling the current and future state of setup times and labor hours on production lines. Next, he calculates the impact on inventory holding costs by modeling the current and future state of cycle stock and safety stock inventory levels. With the information obtained on the costs analysis and a Pareto using sales volumes per market, the SKUs for reduction are chosen. Besides, Hilliard employs SKU portfolio dashboards to monitor SKU development and govern creation of new ones.

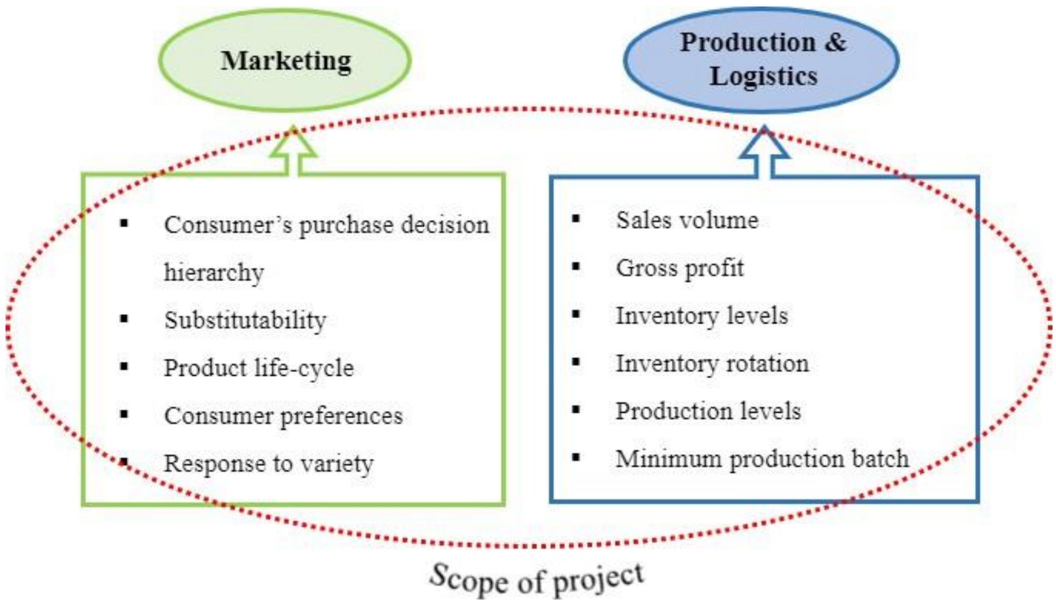
To summarize the literature review reported in this section, Table 1 synthesizes the methodology and the main criteria for SKU rationalization of each reference studied. The criteria are, CP: Consumer preferences, SS: Substitute SKUS, SV: Sales volume, IL: Inventory levels, PM: Profit margins, IR: Inventory rotation, PC: Product Life-Cycle and PL: Production Levels.

Table 1 Summary of literature review (Methodology and criteria)

Reference	Methodology	CP	SS	SV	IL	PM	IR	PC	PL
Bob Byrne (2007)	Study products from the consumer's perspective, identify which SKUs serve unique channels and conduct a failed-product elimination process	x	x						
David Hilliard (2012)	Select the brand and SKU to eliminate, analyze sales and cost impact, monitor and sustainment			x		x			
Dave DeWalt (2005)	Analyze Data and identify candidates. Then evaluate and develop action plans. Following is to project the P&L impact. At last decide, communicate, and implement			x					x
Mittal, Sharma, Batra and Maheshwari (2012)	Define the possible SKUs to be rationalized, make a detail implementation plan, execute the rationalization of the SKUs and finally measure the benefit with one of the three suggest mechanism.			x		x	x	x	
David A. Fields (2006)	Identify the consumer purchase decision hierarchy. Identify the substitutability of the SKUs. Determine the optimal number of SKUs (diminishing returns). Determine sales and profit productivity. Finally, SKU selection.	x		x		x		x	
Capegmini (2007)	First it must be calculated de GMROI identifying the particular needs of the business. Then identify in which category is the GMROI: consumer staples, family jewels, rationalization opportunity or niche performs. After the categorization, make a strategy based on it. Next step is differentiating products with high potential of those with low potential and finally rationalize.			x		x	x		
Michael Gilliland (2011)	Create a Pareto chart ranking the revenue (or volume) of all the product offerings and analyze aspects such as substitute products, inventory levels, among others.		x	x	x				
Barry Berman (2011)	Classify goods into consumer behavior-based tiers; use inter-functional product pruning teams; practice mass customization, placing absolute limits on product choice; and implement effective strategies for product pruning.	x	x		x				

The contribution of this project to the aforementioned methodologies, is the approach of a method to transform the qualitative information into quantitative information. Once all the information is quantitative, it is used to carry out the rationalization. In order to have a wider view of the company it is necessary to take into account at least one qualitative criterion of marketing and one quantitative criterion of production & logistics. In Figure 1 is presented a classification of the criteria according to the area of the company they belong.

Figure 1 Classification of criteria according to the areas of the company



3. Objectives

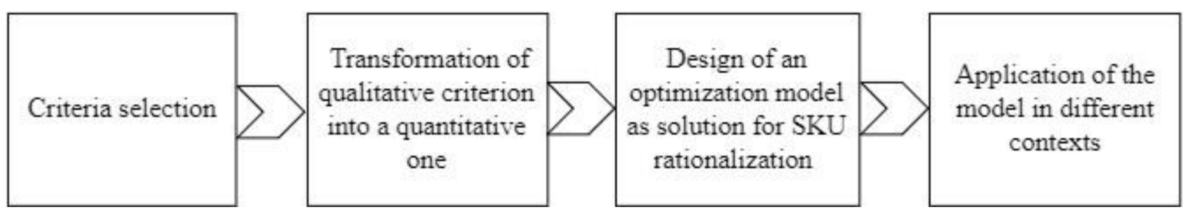
The general objective of this project is *to design a quantitative approach for SKU optimization.* This main objective will be achieved by accomplishing the following specific objectives:

a. Select the qualitative and quantitative criteria to be used within a SKU optimization approach.
b. Design and implement a quantitative method for SKU optimization.
c. Validate and measure the economic impact of the proposed approach based on a case study.

4. Methodology

In Figure 2 we present the methodology implemented for this poject

Figure 2 Methodology



4.1. Criteria selection

We made a complementary literature review for the criteria selection, the results of this search are summarized in Table 2. In the criteria summary table there are some information about each criterion, such as: a brief description, the area of the company it belongs, the authors who wrote about it, the type of variable needed if it would be measured, the information source to measured it and the desired levels.

Table 2 Criteria summary table

	Description	Area	Type of variable	Source	Desired levels	Literature review
Consumer preferences	Products that, from the customer's point of view, best meet their needs	Marketing	Qualitative	Surveys, Interviews	Very High	Byrne, 2007 Hilliard, 2012
Substitutability	Grade of willingness of consumers to replace the product with another	Marketing	Qualitative	Interviews, focus groups, panel of experts	Low	Byrne, 2007 Fields, 2006 Gilliland, 2011 Berman, 2011
Sales volume	Number of units sold in a year	Marketing	Quantitative	Sales history	Infinite	Hilliard,2012 DeWalt,2005 Mittal et al., 2012 Fields,2006 Capegmini, 2007 Gilliland, 2011
Inventory average	Quantity of units in inventory at a given time	Logistics	Quantitative	Inventory levels in the past years	Zero	Fields,2006 Gililland, 2011 Berman,2011
Average Revenue	Difference between the amount earned the amount spent on a certain SKU	Finance	Quantitative	The difference between the sale price and all the costs	Infinite	Hilillard, 2012 Mittal et al., 2012 Fields, 2006 Capegmini, 2007
Inventory rotation	Number of renewal cycles of inventory in a year	Logistics	Quantitative	ROI	Very High	Mittal et al., 2012 Fields, 2006 Capegmini, 2007
Inventory Costs	Cost of keeping one unit of the product in inventory	Logistics	Quantitative	In a year: Holding costs, lot sizes, demand	Zero	Fields, 2006
Operating Costs	Cost of making a unit of a certain SKU	Logistics	Quantitative	Fixed and variable costs for a production lot	Zero	Byrne, 2007
Seasonality Factor	Repercussion of the time of year on the demand of a certain SKU	Marketing	Qualitative	Sales history	Non existent	Van Kampen et al., 2012
Variable Contribution Margin	Difference between the amount earned the amount spent, without taking into account fixed costs, on a certain SKU	Finance	Quantitative	The difference between the sale price and all the variable costs	Infinite	DeWalt, 2005
Shelf life	Length of time that a certain SKU may be stored without becoming unfit for sale	Marketing	Quantitative	Dispatch orders	Near Zero	Mittal et al., 2012
Distribution channels	Different ways on which the product arrives to the consumer	Logistics	Qualitative / Quantitative	Number of distribution channels	Plenty	Byrne, 2007
Sales percentage	The contribution of the sales of certain SKU compared with total sales of the products	Finance	Quantitative	Sales history	High	Hilliard, 2012
SKU complexity	The number of SKUs in a product family, that come from the same mother product	Logistics	Quantitative	Family of products	Near Zero	McCord, Novoa, 2015

Once the criteria revision was made, the next step was to select the criteria to be used. While selecting the criteria, certain empirical theories started to gain importance in the decision-making process, most of them lead to simplify criteria into one criterion, due to the correlations between them. For example, the relationship between inventory levels, inventory rotation and sales volumes. It is believed that, with the proper inventory politics, the inventory rotation will be able to maximize sales while keeping inventory rotation high. Dubelaar, Chow and Lars (2001) proved that there is a significant link between inventory, sales and service level (availability). Likewise, these authors found that consumer demand (consumer preference) appears to impact retail inventory in a predictable manner.

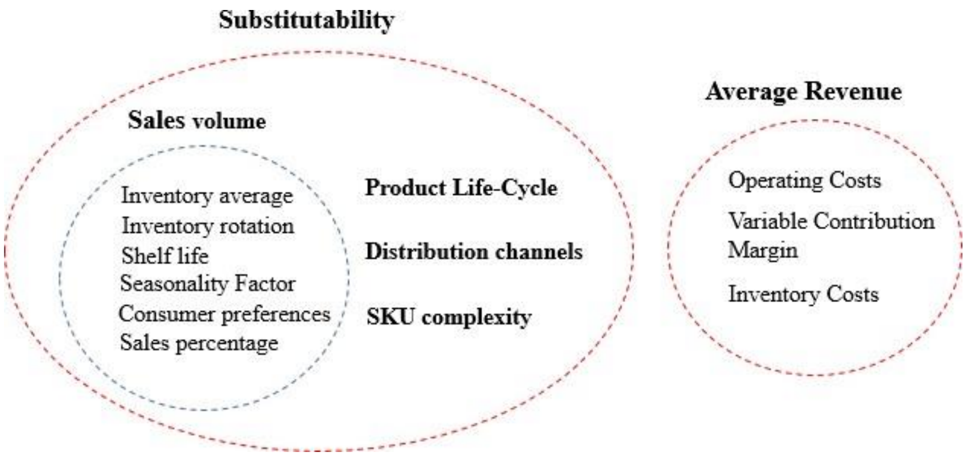
Next in order, it became evident that working with a unique financial unit, would simplify the calculation of the objective function in the optimization model. The Average Revenue per Unit is the total revenue divided by the number of units sold. Is different from price if the producer sells same units from a given product at different prices. Because of this, average revenue per unit is a financial unit that encloses the income related to each product and all the costs involving an SKU. (Ghodke, N. B, 1985). These costs include the fixed and variable costs. The fixed costs enclose administrative and management expenses and maintenance expenses such as insurance, rent, salaries, equipment. (Briciu, Sorin, 1918). The variable costs include operating costs, supplies, promotion and marketing and inventory costs, this last one depends on the inventory policy of each company (Edori, Daniel. 2018).

Another theory that gained force while selecting the criteria was the relationship between substitutability and various criteria from marketing, logistics and production. For instance, Karakul and Chan (2008) affirmed that if substitutability of products is not considered as an integral part of the inventory control and pricing strategies, it is challenging to generate accurate demand forecasts and to determine the right production quantities and price that maximize the profits. On the other hand, McGuire and Staelin (1983) asserted that, if product substitutability is low, the manufacturer will design an integrated distribution channel, but, if product substitutability is high, the manufacturer will design a decentralized channel, further cementing the importance of the substitutability.

Along with this, there is also a relation between the substitutability and the SKU complexity, due to the fact that there are some products that can be the “mother” of many “children products”, giving birth to the SKU proliferation problem. Consequently, as Fields (2006) points out, the reason for SKU rationalization is that numerous items on the shelf are essentially interchangeable. Moreover, as Karakul and Chan (2008) stated, there is a relationship between product substitutability and the product life-cycle, since considering substitution in the initial product introduction helps the firms set a price higher and collect higher profits over the life-cycle of such products.

Given all relations described above, substitutability was selected as the main qualitative criterion to bear in mind in the design of the SKU optimization method. Besides all the benefits described before, this criterion allows to measure the impact when a certain SKU is eliminated. In addition, the average revenue was selected as the main quantitative criterion because it would quantify the financial consequences of eliminating or keeping certain SKUs in relation with the substitutability relationships of those remaining. Figure 3 depicts the summary of the organization of the criteria baseline.

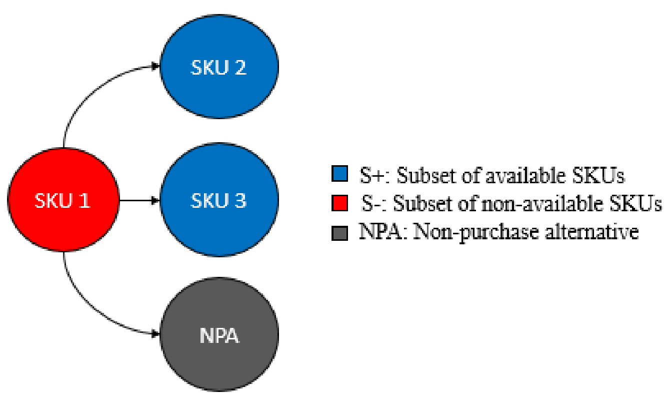
Figure 3 Final organization of the Criteria



4.2. Transformation of substitutability into a quantitative criterion

According to the previous section, the criteria to rationalization are substitutability and average revenue. However, the substitutability is a qualitative criterion, hence it is necessary to transformed it into a quantitative one in order to use it in the optimization method. The substitution relationship between SKUs is difficult to measure, and it is basically a case by case scenario. For example, as shown in Figure 4, there is a portfolio of three products, product 1 is not available and it is known that products 2 and 3 are substitutes of product 1. If customer A is looking for product 1, the probability of switching to product 2 is higher than probability of switching to product 3, or switching to not purchase anything; but that may not be the case for customer B. If customer B is looking for product 1 and it is not available, the probability to not purchase anything is higher than the probability of switching to product 2 or to product 3.

Figure 4 Substitutability example for three products



Therefore, it is necessary to design a substitution behavior model that indicates the probability that a customer, who initially wants to purchase the non-available product 1, finally purchase another product, assuming that the latter is available. Likewise, the model must indicate the probability of the non-purchase alternative.

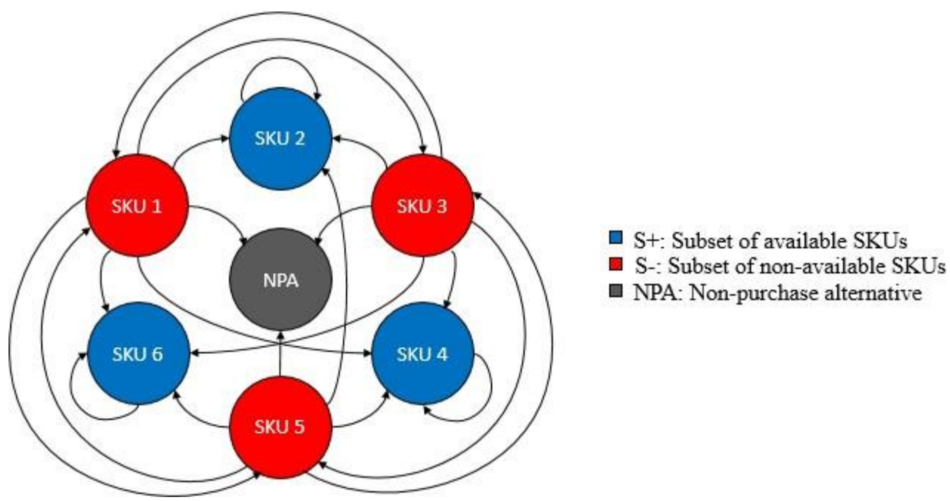
Customer behavioral models have already been developed from the implementation of qualitative research techniques such as focus group, surveys, panel of experts, among others. The output of this researches is the transition probability between two products or between a product and the non-purchase alternative. With that information we proposed to build a matrix where every SKU is crossed against other products and the non-purchase alternative. In this matrix, called the *substitutability matrix* M, the m_{ij} position denotes the probability that a customer who initially is looking for SKU i, ends up choosing SKU j, given that i is not available and j is available (except when $i = j$). It is evident that each row sums 1, which means that in case of eliminating a certain SKU, there are no other options than to substitute the product with any of the other ones or to select to the non-purchase alternative. Clearly, the intersection between a SKU and itself is zero, as it is impossible to substitute an SKU with itself. Below, in Table 3, there is an example of a substitutability matrix for a four-product portfolio. For instance, in this example there is a probability of 0,1295 that a person who initially wants the product 1 finally purchases the product 2 (m_{12}) given that product 1 is not available.

Table 3 Substitutability matrix example

	1	2	3	4	Non-purchase alternative
1	0	0,1295	0,2446	0,3131	0,3128
2	0,2565	0	0,4587	0,1301	0,1547
3	0,1689	0,2068	0	0,1467	0,4776
4	0,1011	0,0788	0,5568	0	0,2633

Nevertheless, product substitution does not often stop at the first transition. When the substitute is not available either, the customer has to go through different alternatives before purchasing a product or deciding to purchase nothing. Figure 5 shows an example of a purchase choice model for a customer who wants to buy a product from a six SKU portfolio and just the products 2, 4 and 6 are available. Let's assume the customer's most preferred product is product 1 but it is not available, so customer decides to substitute it with product 5 but it is not available either, so customer chooses to purchase product 4 that it is available. In this case the customer had to do two transitions, first one to product 5 and second one to product 4. The transitions would finish when the customers find an available product or when the non-purchase alternative is chosen.

Figure 5 Functionality within a substitutability matrix for a six SKU problem



It is worth clarifying that the probability that a customer substitutes product 1 with product 2 directly (in one transition) is different from the probability that customer substitutes product 1 with product 2 after pass through several transitions. Consequently, for the case where there are more than one non-available SKU the substitutability matrix would not contain the final substitution probabilities, as the probabilities shift with each eliminated SKU. That is why it is necessary to find a method to determine where the decision process converges and in this way obtain the final choice probabilities taking into account the transitions between the products.

After a literature review, the substitution model posed by Blanchet, Gallego and Goyal (2013) was selected to determine the final choice probabilities from the substitutability matrix. In this method, the substitution behavior is described as follows: a customer arrives to the shop and selects his/her most preferable product if it is available, if it is not, the consumer will select the second one preferable and so on, taking into account the non-purchase alternative. Therefore, the selection process is interpreted as sequential transitions from one product to another until the customer finds an available product or until the non-purchase alternative is selected.

Having in mind that originally Blanchet, Gallego and Goyal developed the model with the objective of defining the correct assortment to retailers, we had to adapt the model to our problem, the product portfolio rationalization for companies with SKU proliferation. Moreover, it was necessary to perform all the mathematical deduction again, firstly because it was not explicit in the authors' paper and secondly to understand the assumptions made by the authors and to verify that the model does represent the substitutability behavior of the customers. Below is a detailed explanation of the model.

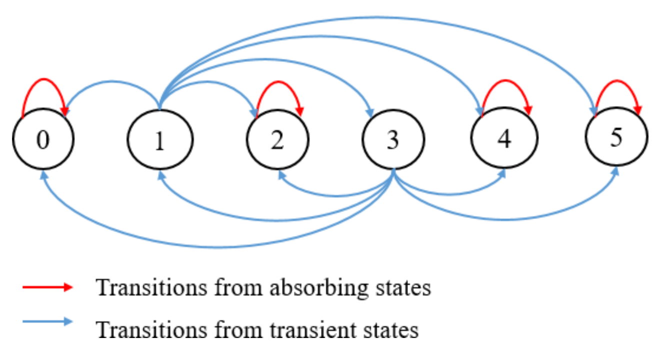
The Blanchet, Gallego and Goyal's model consists of a Markov chain, where there is a state for each product and one for the non-purchase alternative. There is an arrival probability denoted by λ_i than can be interpreted as the arrival rate of customers who prefer the SKU i when everything is offered and there is a transition probability matrix P where p_{ij} is the probability that a customer selects the alternative j when he/she initially wants the i product, so

$$p_{ij} = \{ \begin{matrix} 1 \; when \; i = j \; and \; the \; product \; i \; is \; available \\ 0 \; when \; i \neq j \; and \; the \; product \; i \; is \; available \\ p_{ij} \; = m_{ij} \; when \; i \; is \; not \; available \end{matrix} \}$$

Evidently, the substitutability matrix presented before is related to the transition matrix, due to $m_{ij} = p_{ij}$ when the product i is not available but j is available. However, when i is available there is no relation between these two matrixes because if the most preferred product is i and it is available then the customer would purchase it and the probabilities of switching to other products will be zero. The sum of the arrival probabilities of all SKUs is equals to 1 and the arrival probability to the non-purchase alternative is zero, working under the assumption that at the beginning, the customer always wants to buy one of the products offered in the portfolio. The parameters λ_i and p_{ij} are estimated from the consumer preference.

Now, for each possible assortment, or product portfolio in this case, there is a different Markov chain where all the states corresponding to the available SKUs are absorbing. For instance, Figure 6 shows the Markov chain if there is a portfolio of five products and the SKUs 1 and 3 are eliminated, so they are not available.

Figure 6 Markov chain for a five SKU portfolio



In the previous Markov chain, the 0 state represents the non-purchase alternative, this state and the available SKUs 2,4,5 are absorbing states while the products 1 and 3 are transient states. Note that for every transient state, there is a transition probability to state 0 corresponding to the non-purchase alternative in which case, the customer leaves the system. To know what is the final customer choice, it is necessary to determinate the stationary distribution over all absorbing states, including the non-purchase alternative. Clearly, the Markov chain is an absorbing one, since it is true that:

- The chain has at least one absorbing state
- It is possible to go from any non-absorbing state to an absorbing one

Let P represents the canonic matrix where:

$$P = \begin{bmatrix} I & 0 \\ R & Q \end{bmatrix}$$

Where:

S : Set of all the SKUs
$S^+ \subseteq S$: Subset of available SKUs (absorbing states)
$S^- \subseteq S$: Subset of non-available SKUs (transient states)
I: $(|S^+| + 1) \times (|S^+| + 1)$ Identity matrix
R: $|S^-| \times (|S^+| + 1)$ Submatrix concerns the transition probabilities from transient states to absorbing ones
Q: $|S^-| \times |S^-|$ Submatrix concerns the transition probabilities between transient states
0: $(|S^+| + 1) \times |S^-|$ Submatrix of zeros

To clarify, the expression $|S^+| + 1$ refers to the amount of absorbing states, including the available SKUs plus the non-purchase alternative.

Regardless of the initial state, in a finite number of steps the chain will go into an absorbing state, therefore in the transitory states the limit distribution probability is 0.

$$\lim_{n \to \infty} Q^n = 0$$

Equation 1 Limit distribution probability of Q matrix

Now, for any absorbing Markov chain, the fundamental matrix F is the inverse of $I - Q$:

$$F = (I - Q)^{-1} = I + Q + Q^2 + \cdots = \sum_{k=0}^{\infty} Q^K$$

Equation 2 Fundamental matrix

The fundamental matrix gives the number of visits to state *j* that are expected to occur before the absorption, given that the current state is *i*. Note that $Y = FR$ is a $|S^-| \times |S^+|$ matrix where Y_{ij}denotes the probability that the process starting in transient state *i* ends up in absorbing state *j*.

Once the stationary distribution over all absorbing states is calculated, it is possible to estimate the final choice probability of the available SKUs and the total fraction of the market that selects the non-purchase alternative. The choice probability is constituted by the arrival probability to the absorbing state j plus the probability that the process starting in the transient states *i* ends up in *j*. Then, for all SKU $j \epsilon S^+$ the final choice probability is given by

$$\pi_j = \lambda_j + (\lambda(S^-))^T Y e_j$$

Equation 3 Choice probabilities formula

where $\lambda(S^-)$ is the vector of arrival probabilities in S^- and e_j is the j^{th} unit vector.

As mentioned before, each possible portfolio has a different Markov chain and consequently different final choice probabilities π_i calculated from the λ_i and p_{ij} parameters. The **Algorithm 1** shows the pseudocode we developed to calculate the choice probabilities to a given SKU portfolio

Algorithm 1 Choice Probabilities

begin

Identify the available and non-available SKUs

Create the C and R matrices

Calculate the matrix operation: I – Q

Calculate the fundamental matrix

Calculate the Y matrix

Calculate the final choice probability for each available SKU

end;

4.3. Design of a quantitative approach for SKU optimization

In order to find the optimal assortment, the revenue and the cost of maintaining each SKU must be known. Nevertheless, is too hard to gather that information because would be necessary to consider all the direct and indirect costs of maintaining each SKU in the portfolio. If it were possible, the objective function of the rationalization problem would be the following

$$\max z: \sum_{i=0}^{S} (g_i * \pi_i) - (C_i * x_i)$$

Equation 4 Objective function of the rationalization problem

where

$g_i =$ Expected average revenue of maintaining the SKU $i \in$ S in the portfolio
$\pi_i =$ Final choice probability of the SKU $i \in$ S
$C_i =$ Cost of maintaining the SKU $i \in$ S in the portfolio

and x_i is the decision variable

$$x_i \left\{ \begin{matrix} 1 \text{ } if\ the\ SKU\ i \in \text{S}\ remains\ in\ the\ portfolio \\ 0 \text{ } if\ the\ sku\ i \in \text{S}\ is\ eliminated \end{matrix} \right\}$$

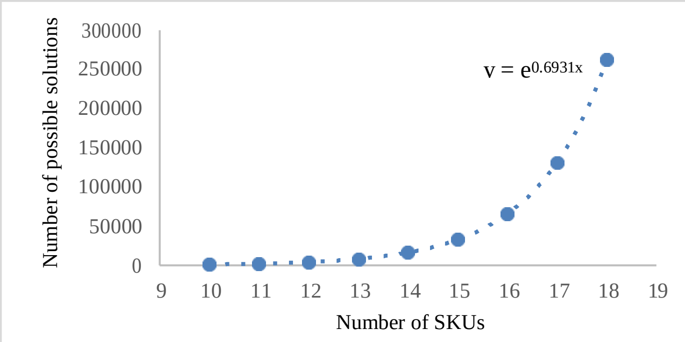
However, as mentioned before it is unlikely that companies with SKU proliferation have the financial information required to implement the previous model, due to the cost and complexity of gathering the data. For that reason, in this project, we proposed an alternative approach where the companies can provide or not input data for the model according to their expectations and requirements. In case that the company do not provide any information, the model will work under some assumptions based on a parametric analysis. In the model proposed, the substitutability and the average revenue of SKUs guide the portfolio rationalization. Considering those criteria, the problem has a huge number of possible solutions. Exactly, to rationalize a portfolio of *m* SKUs there are

$$\sum_{n=0}^{m} C_m^n = \sum_{n=0}^{m} \frac{m!}{n!\,(m-n)!}$$

Equation 5 Number of possible assortments of a portfolio of m SKUs

possible different assortments that must be evaluate, including the possibilities of not eliminating any SKU or eliminate them all. For instance, for a portfolio of 10 SKUs 1024 solutions must be evaluated. Furthermore, it was proved that with each additional SKU in the problem, the amount of solutions growths exponentially as shown in Figure 7. Because of this, an exhaustive search is not tractable, so it is necessary to implement another optimization technique.

Figure 7 Computational time growth by increase the SKUs number



On the other hand, the computation of π_j requires a matrix inversion where the coefficients of the matrix depend on each possible portfolio, hence, there is not a simple form to express the choice probabilities as a function of the parameters λ_i and p_{ij}. Given that the substitutability makes the objective function and constraints nonlinear, the problem cannot be resolved by linear programming. In addition, since the problem consists of finding an optimal object from a finite set of objects, it is a combinatorial optimization problem. This class of problems is characterized by discrete decision variables and a finite search space. Considering all of the above, it was concluded that this is a NP-hard problem. For this class of problems, metaheuristics are one of the competing algorithms to obtain good solutions for instances considered too complex to be solved in an exact manner. Although metaheuristics do not guarantee to find global optimal, they deliver satisfactory solutions in a reasonable time (Talbi, 2009).

Tabu Search (TS) is a metaheuristic that guides a local heuristic search procedure to explore the solution space beyond local optimality. This method is based on the premise that problem solving, in order to qualify as intelligent, must incorporate adaptive memory and responsive exploration. Considering TS is a good method for designing solution procedures for hard combinatorial optimization problems (Glove 1995), we used it to solve the rationalization problem. The **Algorithm 2** presents the pseudocode of the TS method based on the one proposed by Xhafa, Sánchez, Barolli and Takizawa (2015)

Algorithm 2 Tabu Search

```
Begin
    Compute an initial solution s
    let ŝ ← s
    Reset the tabu and aspiration conditions
    do
        Generate a subset N(s) of solutions such that:
            (the tabu condition is not violate) or (the aspiration criteria hold)
        Choose the best solution s′ ∈ N(s)
        s ← s′
        Update the tabu list
        if improvement(s′, ŝ)
            ŝ ← s′
        end if
    while not termination condition
    return ŝ
end;
```

4.4. Implementation of the optimization method

The optimization method proposed in this project is directed to companies that have the SKU proliferation issue in at least one product family with substitutability relations between the SKUs. For instance, the case of the 500 shampoos at Wal-Mart, 85 30-inch televisions at Circuit City, 80 varieties of pens at Office Depot and

15 versions of a single brand of toothpaste at the local drugstore (Byrne 2007). All these companies need to reduce their portfolio maintaining the most suitable products.

Evidently the main objective of every company is to maximize its revenue, however there are ones that have a specific goal with the rationalization like removing a certain percentage of the portfolio defined by managers. For that reason and taken into account that there are companies that have already determined specific goals and the constraints for the rationalization process, different companies' contexts were defined according to available data.

Next, a description of the defined companies' contexts:

1. Companies that have not done any analysis about the rationalization problem, reason why they do not have constraints. In this case the objective function is maximize an indicator that involves both, the revenue and the number of SKUs removed, in this way it is possible to reduce the portfolio to the maximum without neglecting the loss of revenue.

2. Companies that have already defined a constraint for the rationalization process. Two models were defined according to the companies' constraints and goals:

   A. Revenue maximization removing at least a minimum percentage of SKUs. In this case the objective function is the maximization of the revenue and the constraint is related to the specific percentage of the portfolio that company aims to reduce. This context happens when a minimum number of SKUs to remove is defined, either by the managers or as a result of an external analysis like a benchmarking study.

   B. Portfolio reduction meeting the maximum percentage of revenue that the company is willing to lose with the rationalization. In this context the objective function is to minimize the number of SKUs kept but without having a loss of revenue larger that the allowed by the company.

As the constraints and objective functions of the contexts are different, hence the mathematical model are different too. Regardless the context, the optimization method consists on the implementation of the substitutability model into TS. Below, the parameters and decision variable used are described

Decision variable

$$x_i \left\{ \begin{array}{c} 1 \; if \; the \; SKU \; i \in \mathrm{S} \; remains \; in \; the \; portfolio \\ 0 \;\; if \; the \; sku \; i \in \mathrm{S} \; is \; eliminated \end{array} \right\}$$

It is worth remembering that π_i changes according to each possible assortment and if x_i is zero for a given SKU $i \in \mathrm{S}$ then π_i is zero too.

Parameters

$G =$ Total average revenue generated with the current portfolio
$H =$ Minimum percentage of SKUs to eliminated
$K =$ Maximum percentage of lost revenue
$g_i =$ Expected average revenue of maintaining the SKU $i \in \mathrm{S}$ in the portfolio

In the following comparative table, the model of each context is explained in detail

Table 4 Brief of the companies' contexts

	Context 1	Context 2 - A	Context 2 - B
Objective	Maximize the efficiency rationalization indicator	Maximize the final revenue after the rationalization	Minimize percentage of SKUs that remain in the portfolio
Objective Function	$z: ERI$	$z: \sum_{i=1}^{\lvert S\rvert} \pi_i g_i$	$z: \sum_{i=1}^{\lvert S\rvert} \frac{x_i}{\lvert S\rvert}$
Constraint	There are not constraints in this model	There is a minimum percentage of SKUs to eliminate $1 - \sum_{i=1}^{\lvert S\rvert} \frac{x_i}{\lvert S\rvert} \geq H$	There is a maximum percentage of lost revenue tolerated $1 - \frac{\sum_{i=1}^{\lvert S\rvert} \pi_i g_i}{G} \leq K$
Target company	Companies who have not defined how many SKUs to eliminate neither how much revenue are willing to lose	Companies who have already defined a percentage of SKUs to eliminate	Companies who have not defined how many SKUs to eliminate, but have already determined how much revenue are willing to lose

Taken into account that in context 1, the company do not have constraints for the rationalization, the ***efficiency rationalization indicator*** *(ERI)* concept is developed. The ERI aims to include both, the percentage of SKUs eliminated and the percentage of final revenue after the rationalization

$$ERI = \alpha\,\left(1 - \frac{\sum_{i=1}^{|S|} x_i}{|S|}\right) + \beta\,\left(\frac{\sum_{i=1}^{|S^+|} \pi_i g_i}{G}\right) \; where\ \alpha + \beta = 1$$

Equation 6 Efficiency rationalization indicator expression

Evidently, the context 1 is a multi-objective problem. For this kind of problems, the weighted sum method provides a basic and easy to-use approach. Transforming the functions so that they all have similar magnitudes allows the weights to reflect preferences more accurately because there is not a natural dominance into the objective function (Marler & Arora, 2010). For that reason, in the ERI both objectives are in terms of percentage and not in terms of number of SKUs and reimaging revenues.

Furthermore, it was theorized that it was necessary to implement the weights α and β because if both objectives had the same weights the results obtained, although feasible, it could not be implemented in a real context. For instance, without weights it is possible to obtain an ERI of 1,1 where the percentage of SKUs to eliminate is 80% and the percentage of remaining revenues is 30%. In that case, although the percentage of SKUs eliminated is very high, the percentage of lost revenue also is, in fact, so high that probably the company will not be willing to assume that huge lost. Hence, with the implementation of the weights it is possible to give a bigger importance to the revenue and the company can test different weights' values until getting a satisfactory solution.

The ERI formula is a simplification of the model originally presented in the section 4.3 where the company has all the required financial information. By establishing weights in the terms of the ERI formula, we provide the model the information that the company does not have, that means, by defining a value α and β we are establishing the cost of maintaining each SKU in the portfolio. In order to establish suitable weights, the model was tested with different weights' values and a parametric analysis was done.

4.5. *Simulation of the case of study*

Owing to the high amount of necessary resources to gather and analyze the market data, nowadays few companies have already defined the substitutability behavior of their customers. For that reason, in order to implement the method proposed in a case of study, the substitutability matrix (input data) was simulated. With the purpose of considering different kind of the portfolios, four SKU categories were defined based on two criteria, the preference and the substitutability. Products classification is shown in Table 5. The preference implies that consumers have a definitive ranking between the options allowing them to know whether one alternative is at least as good as the others (Dhar 1997). A high preference level indicates the product tends to be chosen by a customer in the purchase decision, while a low preference level means the product is one of the last purchase options for the customer. The fact remains that a company which is interested in optimizing the product portfolio must follow some specific steps to accomplish the rationalization. These steps are shown in the annexes (183002- Annexes Steps to rationalize).

In this model, the substitutability refers to the number of substitute SKUs offered in the same portfolio. Therefore, assuming the selection process as sequential transitions through the substitute SKUs, if the substitutability level of a product is high, the probability of the non-purchase alternative will tend to be lower and vice versa.

Table 5 Products classification

Type of product	Preference	Substitutability
Product 1	High	High
Product 2	High	Low
Product 3	Low	High
Product 4	Low	Low

For a given SKU i the substitutability probability m_{ij} will be greater if the substitute SKU j has a high preference level, consequently, the SKUs type 1 and type 2 will have greater final choice probabilities.

Likewise, the arrival probabilities to each SKU and its respective revenue were simulated. The arrival probability is directly proportional to the preference level, so if the SKU belongs to the category 1 or 2 then the arrival probability will be high and vice versa. In contrast, the revenue does not depend on the type of product. Both SKUs with high preference level and low level could have high and low revenue; hence, the revenue is generated randomly.

The **Algorithm 3** presents the pseudocode we proposed for generating the substitutability matrix and the SKUs attributes

Algorithm 3 Generate substitutability matrix and SKUs attributes

```
begin
    for i =1 to Number of SKUs
        Generate a random revenue of the SKU i
        Generate the arrival probability to SKU i based on the type of product
        Generate the probability to the non-purchase alternative from SKU i based on the type of product
        for j =1 to Number of SKUs
            Generate the substitutability probability m_ij considering the type of product of the substitute SKU j
end;
```

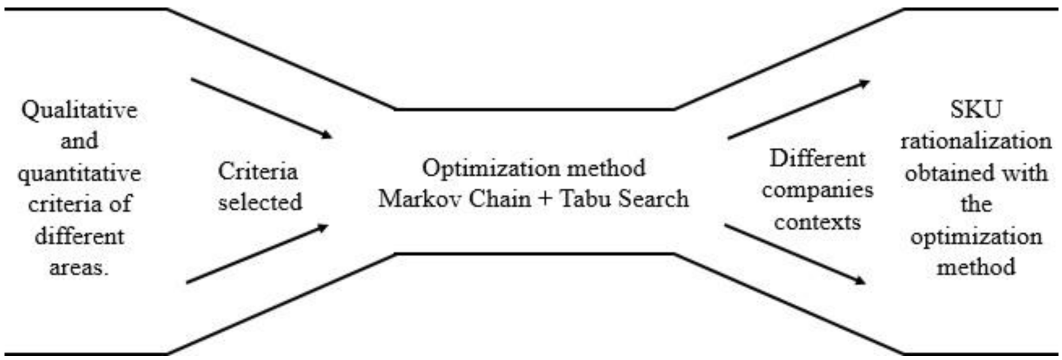
Finally, to explain completely the implementation of the optimization method, the **Algorithm 4** depicts the pseudocode of the computational application designed. This algorithm is the same for all the companies' contexts, the code only varies in calculation of the objective function and the feasibility evaluation according to the context as indicated in Table 4.

Algorithm 4 General structure of the models

begin
call Generate substitutability matrix and SKUs attributes algorithm
call Tabu search algorithm
end;

To finish with this section, below, Figure 8 depicts the design process of the rationalization method

Figure 8 Evolution of the methodology



According with the methodology presented at the beginning of this section, the design process consisted in the selection of a quantitative criterion and a qualitative criterion, and the implementation of a Markov Chain in TS metaheuristic to solve the combinatorial problem. Subsequently, the method was adapted to the contexts of several companies to consider different objectives and inputs for the portfolio rationalization.

5. Results

The method proposed was implemented in different scenarios based on a fictitious company which showcases the SKU proliferation issue. In order to have a diverse portfolio, 60 SKUs were considered, 15 per each category. The first 15 SKUs belong to the first category, the following 15 to the second one and so on. The subsequent problem is then defined as follow:

In recent years, the manufacturing company XYZ has been growing quickly due to the high innovation and investment in the product development area. Nevertheless, as the number of SKUs increases due to the product customization, managers realize that not all products are profitable. On the contrary, having a large amount of products has increased the costs and complexity across the supply chain. Therefore, they decided to reduce the portfolio. Company XYZ is interested in rationalizing specifically a family of 60 products. As a market study determined, these products have a strong substitutability relationship among them. Based on this study, the company built the substitutability matrix and estimated the probability that customers prefer each product of the current portfolio.

First context

The managers of the company XYZ are very interested in reducing the product portfolio; however, they have no idea about the rationalization process. For that reason, they want to have a first approach without having to provide any constraint. In this situation companies use a first approach to SKU rationalization to view an extreme outcome coming from the process, in order to gain familiarity with the results and have an idea about the outcome.

Next, in Table 6 the results obtained for this scenario are presented

Table 6 Results of the first context

Total revenue before the rationalization	$1.989.348
Total revenue after the rationalization	$1.960.010
Amount of SKUs removed	30
Percentage of lost revenue	1,47%
Percentage of SKUs removed	50,0%

Given that with the ERI both, the revenue and the number of removed SKUs are considered, it is possible to achieve suitable results for both objectives. For instance, in this case half of the portfolio was removed with only a loss of revenue of 1,47%. Such huge rationalization is possible because the most profitable products are kept, therefore customers who prefer unprofitable products now are buying profitable ones.

The amount of removed SKUs per category is in the next table

Table 7 Amount of SKUs removed per category in context 1

Category	Amount of SKUs removed	Percentage
1	9	30%
2	0	0%
3	14	47%
4	7	23%
Total	30	100%

As shown in Table 7, the SKUs of the third category are the most attractive to be removed due to their low preference and their high substitutability. In other words, it is unlikely that a person wants a product of third category but in case it happens, and the product is not available; there are many substitutes for it. That is why, almost all the SKUs of this category were removed. On the other hand, the products of the second category are not attractive for the rationalization because they have high preference and low substitutability, so if they are removed a huge part of the current market would be lost, consequently all products of this category were kept. Despite that the products of the fourth category have low preference, some of them were kept due to their large revenue and low substitutability. If they were removed, the loss of revenue would be large because they have few substitutes. So, is better to maintained them and remove products with lower revenue and high substitutability, those were the first category's products.

The results presented before were obtained with weights of α=0.3 and β=0.7 in the ERI formula; it means that a larger importance was given to the maximization of the revenue. Through a parametric analysis (Annexes), in order to test the hypothesis about the importance of differentiated weights for the parameters α and β leaning towards a greater value for β, it was found that sometimes if the weight of the revenue (β) in the ERI is less or equal to the weight of the number of removed SKUs (α), the rationalization results are not applicable for some companies. If α is greater, the priority will be to eliminate as many products as possible, however it is unlikely that companies remove more than half of their current portfolio having a high loss of revenue. That is why β is greater than α.

Second context - model A

After experiencing unprecedented problems (complains from the sellers which cannot store and sell all the products, capacity utilization reaching the maximum levels and the complexity of the supply chain increasing notoriously over time), the board of directors decided to hire a consultancy firm in order to run a high-level diagnosis across the company's fronts to determine the best solution. The diagnosis suggested a rationalization of at least 10% of the SKUs in order to improve the company's situation.

Due to the complexity of the relationships between SKUs the diagnosis did not state which products must be eliminated and which must be kept to have the maximum revenue. Clearly, in this scenario the company

XYZ is in the second context, where some analysis had already been done and it is possible to provide information to the model. In this case is used the model A, where there is a constraint related to the minimum percentage of SKUs to eliminate and the objective is to maximize revenue.

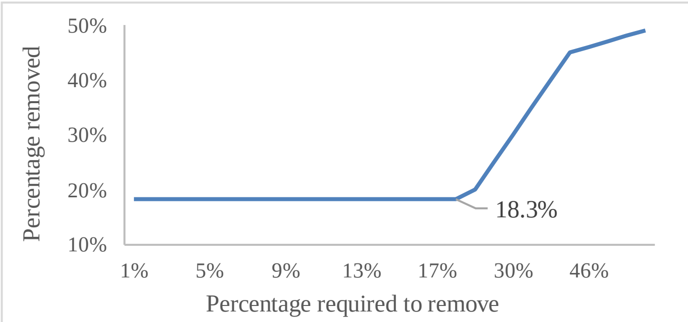
After applying the optimization method, the results were the following

Table 8 Results of the second context – model A

Total revenue before the rationalization	$ 1.989.348
Total revenue after the rationalization	$ 2.072.976
Amount of SKUs removed	11
Percentage of lost revenue	- 4,2%
Percentage of SKUs removed	18,3%

As the results show, the best solution was to eliminate 11 SKUs despite the constraint was to remove at least 6 products (10%). In order to understand better the model's behavior, it was tested with constraints up to 19%, and it was found that the best result was always to reduce 18,3% of the portfolio. However, if the percentage to eliminate is larger than 19% then the model will remove exactly the SKUs required to meet the constraint, as shown in Figure 9

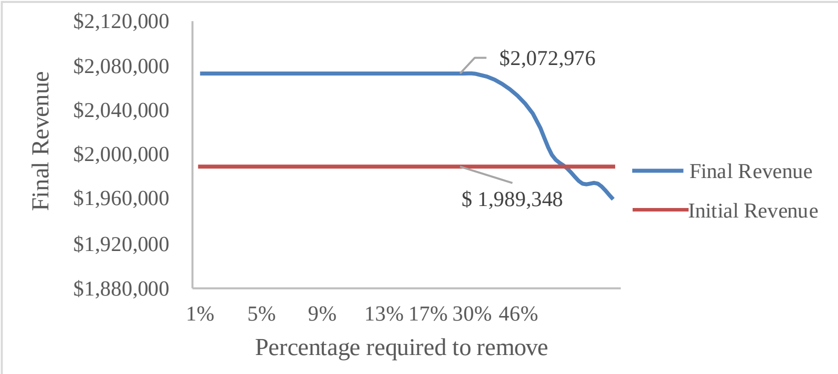
Figure 9 Percentage of removed SKUs according to the percentage required



The explanation for this behavior is the substitutability among the products. The eleven products removed were not profitable, that means, those products had low revenue but high substitutability. So, the best option is to eliminate those products, in that way the customers who preferred them will choose others with high revenue. Consequently, it is possible to eliminate more SKUs than the required as long as a higher revenue is generated. For instance, in this case the revenue increases by 4,2% removing eleven products.

Now, according to the Figure 10, if the percentage to remove is greater than the percentage with which the optimal portfolio is obtained the profits will decrease. In this case, when the constraint is larger than 18,3%, then final revenue will be lower.

Figure 10 Final revenue according to the percentage required



As you can see in previous figure, with this model it is possible to determine the constraint's value from which there is a loss of revenue, in this case when the percentage to eliminate is larger than 46% the final revenue is lower than the initial, then there are losses. It is recommended to companies initially they implement this model with a low constraint of percentage of SKUs to remove, in that way they would find the products that are not profitable and the could increase their revenue.

On the other hand, as shown in Table 9, seven products of the first category and four products of the third category were removed, all of them had lower revenues. It is worth remembering that the first and third category are characterized by the high substitutability level. Due to that characteristic and the low revenue, it is probably that the customers that originally prefer those products choose others with higher revenue. No SKU of second and fourth category was removed thanks to the low substitutability. If these products were eliminated the percentage of the customers that prefers them would probably choose the non-purchase alternative at the end. For all the above, it is concluded that the first candidates to be removed are the SKUs with high substitutability and low revenues.

Table 9 Amount of SKUs removed per category in context 2 – model A

Category	Amount of SKUs removed	Percentage
1	7	64%
2	0	0%
3	4	36%
4	0	0%
Total	11	100%

Second context - model B

In this context, the managers hire a financial consulting agency that made an analysis in which they determined that the maximum revenue loss the company can withstand is 5% of their current revenue. The objective is to eliminate the highest quantity of products meeting the constraint of lost revenue. In this context companies seek to prioritize the best-selling SKUs in order to focus all the efforts into establishing and reinforce these products within the market.

Next, the results of the rationalization for this scenario are presented

Table 10 Results of the second context – model B

Total revenue before the rationalization	$ 1.989.348
Total revenue after the rationalization	$ 1.893.404
Amount of SKUs removed	32
Percentage of lost revenue	4,8%
Percentage of SKUs removed	53,3%

As you can see in Table 10, the number of products removed is considerable, more than half of the portfolio. This happens because the objective in this context is to minimize the number of SKUs kept, consequently the model tries to eliminate all the products, but the constraint related to the lost revenue prevents this from happening. The quantity of SKUs removed per category is shown in Table 11.

Table11 Amount of SKUs removed per category in context 2 – model B

Category	Amount of SKUs removed	Percentage
1	8	25%
2	0	0%
3	13	41%
4	11	34%
Total	32	100%

As in the previous scenarios, it is evident that the model takes as the last option removing SKUs of the second category. Those products have a high level of preference but a low substitutability level. That means that the probability that a customer arrives and wants a product of this category is high, but if it is not available, the probability of finding a substitute is low, so the customer would choose the non-purchase alternative. In this scenario, almost all the SKUs of the fourth and third category were removed, as mentioned before, that happens because they have low preference and revenue.

It is important to take into account that regardless the context, rationalization process depends on the on the magnitude of substitutions probabilities, the preferences and the revenues. Therefore, the assumptions made before are not meet exactly in all cases, given that the parameters of each possible assortment are different and they can influence the result of the rationalization. Nevertheless, the previous analysis allows verifying the utility of the Markov chain into the TB metaheuristic to solve the combinatorial problem and it provides some insights about the way that the method select the SKUs according to the categories.

Additional impact

In addition to the financial impact presented in the aforementioned results, there are other potential impacts of the rationalization. As mentioned in the problem statement, with rationalization several areas of the company are benefited due to the decrease of the supply chain's complexity. For instance, from the point of view of production by removing products it is possible to concentrate the production capacity in the profitable SKUs and at the same time reducing the setup times. On the other hand, the choice process becomes less stressful because now customers have not to sort through a half dozen varieties to find the product, this in turn increases the satisfaction level.

6. Conclusions and recommendations

6.1. Conclusions

- ✓ The method proposed allows to include the customers' choice behavior into the rationalization process. Likewise, with the implementation of the method, it is possible to transform the information about substitutability into quantitative data to be used together with the revenue of the products as the criteria for the optimization.

- ✓ The quantitative approach designed take into account different companies' contexts according to the available information they have. On one hand, there are the companies which have no idea about rationalization, so the method provides them a first insight of the SKUs they can removed to have a better revenue performance. On the other hand, there are the companies that have already done analysis about rationalization and can provide constraints to the model according to their expectations and requirements, in that case the method provides them the best portfolio that meet the constraint defined.

- ✓ By using weights in the ERI formula companies can experiment with different values until finding a relation between α and β that generates a satisfactory solution, instead of calculating the cost of maintaining each SKU in the portfolio. This last method would be a more complex task considering that all direct and indirect costs of each product must be included.

- ✓ Model A of context 2 can be used to find unprofitable products. When these products are eliminated, it is possible to obtain higher revenues than the initial ones, because the customers who preferred them will now choose profitable products. Therefore, this model provides the right portfolio that companies should have if they want to increase their revenues and eliminate that part of the portfolio that is not worth having. After applying this model and eliminating unprofitable SKUs, companies can start a new rationalization process to eliminate a certain required percentage or to eliminate the maximum possible while maintaining at least a certain percentage of revenues.

6.2. Recommendations

- ✓ Due to the limited scope of the project, the main qualitative criterion for the optimization method was the substitutability, but for future studies, it could be considered the complementarity between the SKUs. This criterion refers to the probability that a customer buys a SKU given that he/she purchases another one.

- ✓ As it was mentioned in the results section the arrival and transition probabilities have a high influence on the optimization method, then, it is recommended to invest in market studies in order to estimate correctly the information about the customers' behavior.

- ✓ For future studies, more criteria should be included besides substitutability and revenue, in order to sharpen the model, have solutions more accurate and a wider view of the SKUs' behavior across the supply chain.

- ✓ Nowadays there are multiple ways on which a company can promote its products, bringing the possibility to buy these products from diverse distribution channels (retail store, hard discount stores, e-commerce stores). Thus, we recommend bearing in mind each the different distribution channels in the rationalization process, because the interaction between a customer and the product can be heavily influenced by the channel in which they're involved. It also affects the costs surrounding the product.

7. Glossary

- Optimization Model: Optimization, also called mathematical programming, refers to the study of decision problems in which one seeks to minimize (min) or maximize (max) a function by systematically choosing the values of variables within their allowed sets. An optimization model defines the required input data, the desired output, and the mathematical relationships in a precise manner (Zhang, Lu & Gao 2015).

- SKU Proliferation: Is a problem that began decades ago for consumer products companies as they were looking for a way to respond to the consumer's every need, getting closer to him/her. The result has been a consumer-focused production leading to overwhelming amounts of products with slightly small differences (Byrne 2007).

- SKU Rationalization: According to Quantum retail technology (2010) it's the other side of the SKU proliferation pendulum. Simultaneously Mittal, Sharma, Batra and Maheshwari (2012) defines it as an important technique in inventory management that helps retailers optimize their assortments by decommissioning some of the non- productive merchandise

- Substitutability: Is the characteristic of certain products that, from the consumer's standpoint, are essentially interchangeable (Fields 2006).

- Markov chain: It is a process describing trajectories where successive quantities are described probabilistically according to the value of their immediate predecessors. In many cases, these processes tend to an equilibrium and the limiting quantities follow an invariant distribution (Gamerman 1997).

- Transient state: if the probability of returning is less than one, the state is called transient (Kemeny 1976).

- Absorbing state: A state in a Markov chain is called an absorbing state if once the state is entered, it is impossible to leave (Kemeny 1976).

- Metaheuristics: Are solution methods that orchestrate an interaction between local improvement procedures and higher-level strategies to create a process capable of escaping from local optima and performing a robust search of a solution space (Glover & Kochenberger 2002).

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Annexes

Due to possibility of using different weights in the ERI formula, we performed a parametric analysis with different values to determine the suitable ones to the problem. The chosen scenarios were:

- α =0.3 and β =0.7
- α =0.5 and β =0.5
- α =0.7 and β =0.3.

The results of the first scenario were presented and analyzed in the Results section.

Second scenario was α =0.5 and β =0.5, the results obtained are shown in Table 1

Table 1 Results with α =0.5 and β =0.5 in the first context

Total revenue before the rationalization	$1.989.348
Total revenue after the rationalization	$1.709.181
Amount of SKUs removed	40
Percentage of lost revenue	14,08%
Percentage of SKUs removed	66,7%

In comparison with the first scenario where revenue has priority, in this scenario more SKUs were removed, and the percentage of lost revenue was larger. The number of SKUs removed increased from 30 to 40 and the percentage of lost revenue increased from 1,47% to 14,08%. Clearly that happens because in this scenario the weights of both objectives are equals.

The second additional scenario with weights α =0.7 and β =0.3. The results are the following

Table 2 Results with α =0.7 and β =0.3 in the first context

Total revenue before the rationalization	$1.989.348
Total revenue after the rationalization	$1.275.209
Amount of SKUs removed	49
Percentage of lost revenue	35,90%
Percentage of SKUs removed	81,7%

As weight of the number of SKUs to remove is greater that the revenue´s one, the percentage of removed SKUs grows to 81,7%, meaning only 11 of the 60 SKUs are kept. In the same manner as in the aforementioned assumptions, the SKUS of the third category are the first selected to be removed due to their nature. In this case all SKUs of the fourth category are removed too, since their nature is to have a low preference meaning not too many customers are looking for this type of product. Comparatively almost all SKUs in first category were removed from the product portfolio, that happens because their high substitutability allows to the customer to find another product to satisfy their needs. Just one SKU of the first category was maintain and is because of the high revenue it has.

As you can see in Table 3, the majority of the kept SKUs were from the second category, removing the ones that had the lower revenue. In table # is shown the quantity of removed SKUs per category.

Table 3 Amount of removed SKUs per category with α =0.7 and β =0.3 in the first context

Category	Amount of SKUs removed	Percentage
1	14	29%
2	5	10%
3	15	31%
4	15	31%
Total	30	100%

Because of the results of the parametric analysis, in the model proposed for the first context, it is suggested to use a larger weight for the revenue term in the ERI formula.

Steps a company must follow to accomplish rationalization

To accomplish the rationalization is necessary to follow some steps to ensure company's satisfaction. Next, are the steps the authors propose:

1. Define the product family to be rationalize
2. Identify the aim of the rationalization: first approach to rationalization, maximize the revenue meeting the constraint of minimum number of SKUs to be eliminated, eliminate as much SKUs as possible meeting the constraint of loss revenue.
3. Recognize which of the contexts proposed fits the needs of the company and the model that must be used.
4. Gather the pertinent information to use in the model: average revenue per unit and arrival and transition probabilities
5. Apply the model to obtain the must be kept SKUs and the final indicators associated to the rationalization.

Here can be find some examples or recommendations of each of the steps.

<table>
<tr><th>Steps</th><th>Example/Recommendations</th></tr>
<tr><td>1</td><td>The family product to rationalize must be the one with major proliferation and with substitutability probabilities between its products. Additionally, the maximum average revenue per unit of the products of the family must not be higher than three times of the minimum.</td></tr>
<tr><td>2</td><td>Maximize the revenue meeting the constraint of minimum number of SKUs to be eliminated</td></tr>
<tr><td>3</td><td>After identifying the aim of the rationalization, the company can search in the next table to find the most suitable model:
<table>
<tr><th></th><th>Context 1</th><th>Context 2 - A</th><th>Context 2 - B</th></tr>
<tr><td>Objective</td><td>Maximize the efficiency rationalization indicator</td><td>Maximize the final revenue after the rationalization</td><td>Minimize percentage of SKUs that remain in the portfolio</td></tr>
<tr><td>Objective Function</td><td>z: ERI</td><td>$z: \sum_{i=1}^{|S|} \pi_i g_i$</td><td>$z: \frac{\sum_{i=1}^{|S|} x_i}{|S|}$</td></tr>
<tr><td>Constraint</td><td>There are not constraints in this model</td><td>There is a minimum percentage of SKUs to eliminate
$1 - \frac{\sum_{i=1}^{|S|} x_i}{|S|} \geq H$</td><td>There is a maximum percentage of lost revenue tolerated
$1 - \frac{\sum_{i=1}^{|S|} \pi_i g_i}{G} \leq K$</td></tr>
<tr><td>Target company</td><td>Companies who have not defined how many SKUs to eliminate neither how much revenue are willing to lose</td><td>Companies who have already defined a percentage of SKUs to eliminate</td><td>Companies who have not defined how many SKUs to eliminate, but have already determined how much revenue are willing to lose</td></tr>
</table>
As the aim is to maximize the revenues meeting the constraint of minimum number of SKUs to be eliminates the model chosen is: Second context - Model A</td></tr>
<tr><td>4</td><td>As this information is highly important and must of the companies do not have it immediately, we strongly recommend hiring and external outsourcing to make an appropriate market study.</td></tr>
</table>

5	see left table below	see right table below

Total revenue before the rationalization	$ 1.989.348
Total revenue after the rationalization	$ 2.072.976
Amount of SKUs removed	11
Percentage of lost revenue	- 4,2%
Percentage of SKUs removed	18,3%

Category	Amount of SKUs removed	Percentage
1	7	64%
2	0	0%
3	4	36%
4	0	0%
Total	11	100%